Introduction to Magnetohydrodynamic Modeling of the Magnetosphere

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Outline

- Brief history of global modeling
- Numerical Issues related to MHD Modeling
  - Form of the Equations
  - Computational Methods
  - Parallel Computing
- Global Magnetospheric Modeling Issues
  - Grids and Boundary Conditions
  - MI Coupling
- Validation
- Conclusions and Future Directions

A Brief History of Global MHD Simulations

- 1978 – First 2D simulations by Leboeuf et al.
- Early 80s – First 3D simulations by Brecht, Lyon, Wu and Ogino
- Late 80s – Model refinements including FACs, ionospheres, higher resolution
- 90s – ISTP integrates theory and modeling with spacecraft missions and comparisons with *in situ* space and ground observations begun
- Today – Global modeling has become integrated part of many experimental studies and we've begun linking models of different regions together

Ideal MHD Equations
Non Conservative Formulation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left(\rho \vec{u} \right) = 0$$

$$\rho \frac{\partial \vec{u}}{\partial t} + \rho \left(\vec{u} \bullet \nabla \right) \vec{u} + \nabla \left(p + \frac{B^2}{8\pi} \right) + \left(\vec{B} \bullet \nabla \right) \vec{B} = 0$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \times \left(\vec{u} \times \vec{B} \right) = 0$$

$$\frac{d}{dt} \left(\frac{p}{\rho^{\gamma}} \right) = 0$$

$$\nabla \bullet \vec{B} = 0$$

- No strict numerical conservation of energy and momentum
- Various numerical issues
  - Errors in propagating strong shocks
  - Errors in RH Conditions
  - Incorrect shock speeds

Ideal MHD Equations
Full Conservative Formulation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left[\rho \vec{u} \right] = 0$$

$$\frac{\partial \left(\rho \vec{u} \right)}{\partial t} + \nabla \bullet \left[\rho \vec{u}\vec{u} + \left(p + \frac{B^2}{8\pi} \right) \overleftrightarrow{I} + \frac{1}{4\pi} \vec{B}\vec{B} \right] = 0$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \bullet \left[\vec{u}\vec{B} - \vec{B}\vec{u} \right] = 0$$

$$\frac{\partial \left(\rho E \right)}{\partial t} + \nabla \bullet \left[\vec{u} \left(\rho E + p + \frac{B^2}{8\pi} \right) - \vec{B} \left(\vec{u} \bullet \vec{B} \right) \right] = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$p = (\gamma - 1) \left[E - \frac{1}{2} \rho u^2 - \frac{1}{2} B^2 \right]$$

- Strict numerical conservation of mass, momentum and energy
- Numerical difficulties in low β regions
  - negative pressures possible because p becomes difference of two large numbers

Ideal MHD Equations
Gas Conservative Formulation

$$\frac{\partial \rho}{\partial t} + \nabla \bullet \left[\rho \vec{u} \right] = 0$$

$$\frac{\partial \left(\rho \vec{u} \right)}{\partial t} + \nabla \bullet \left[\rho \vec{u}\vec{u} + \left(p + \frac{B^2}{8\pi} \right) \overleftrightarrow{I} + \frac{1}{4\pi} \vec{B}\vec{B} \right] = 0$$

$$\frac{\partial \vec{B}}{\partial t} + \nabla \bullet \left[\vec{u}\vec{B} - \vec{B}\vec{u} \right] = 0$$

$$\frac{\partial \left(E_p \right)}{\partial t} + \nabla \bullet \left[\vec{u} \left(\frac{\rho u^2}{2} + \frac{\gamma}{\gamma - 1} p \right) \right] + u \bullet \nabla \bullet \left[\frac{B^2}{8\pi} \overleftrightarrow{I} - \frac{\vec{B}\vec{B}}{4\pi} \right] = 0$$

$$\nabla \bullet \vec{B} = 0$$

$$p = (\gamma - 1) \left[E_p - \frac{1}{2} \rho u^2 \right]$$

- Strict numerical conservation of mass, momentum and plasma energy
  - Final term in Energy equation is $\vec{j} \cdot \vec{E}$ dotted with velocity
  - no strict conservation of total energy
- No difficulties in low β regions

Time Differencing

- Explicit time differences
  - Predictor – Corrector (2nd order accurate)

$$U^{n+\frac{1}{2}} = U^n - \frac{1}{2}\Delta t \nabla \bullet F(U^n)$$

$$U^{n+1} = U^n - \Delta t \nabla \bullet F(U^{n+\frac{1}{2}})$$

  - Leap Frog Scheme (2nd order accurate)

$$U^{n+1} = U^{n-1} - 2\Delta t \nabla \bullet F(U^n, U^{n-1})$$

- Stability Criterion (CFL Condition)

$$\Delta t_{\max} \le \frac{\min(\Delta x)}{v}$$

- Implicit Schemes generally not used because soloution of large linear systems becomes too expensive

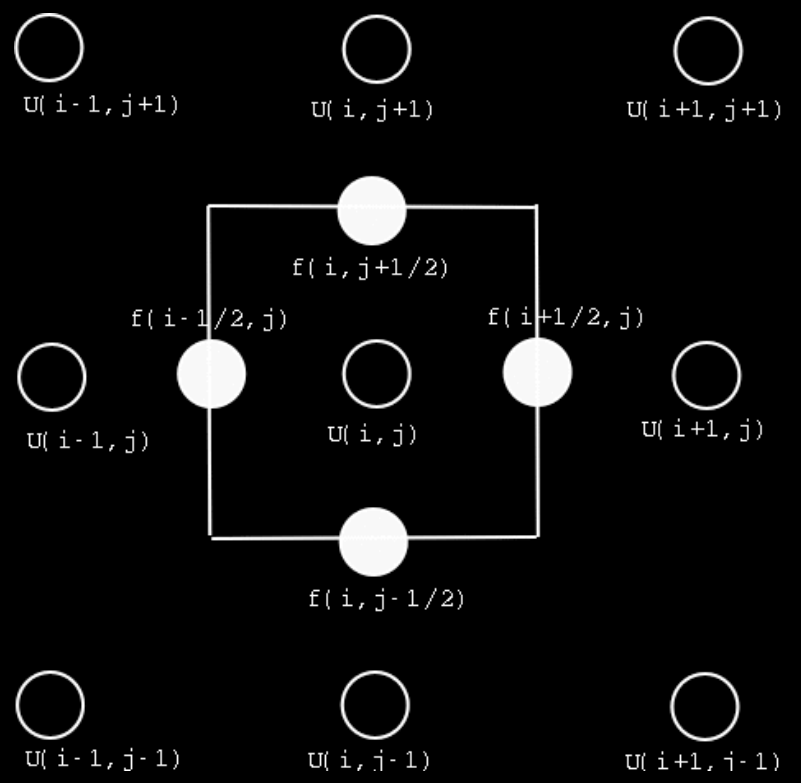
MHD Numerics

- Need a method to solve the conservative formulation of the MHD equations which maintains the conservation properties
- For this disucssion we'll consider the linear advcetion equation

$$\frac{\partial U}{\partial t} + \nabla \bullet F(U) = 0$$

$$\frac{\partial U}{\partial t} + v\frac{\partial U}{\partial x} = 0$$

Spatial Discretization



Conservative Finite Difference Scheme

- State variables are cell centered quantities and we discretize our model equation with numerical fluxes through the cell interfaces

$$\frac{dU}{dt} = -(f_{i+\frac{1}{2}}(U) - f_{i-\frac{1}{2}}(U)/\Delta x$$

- Scheme is conservative because

$$\frac{\partial}{\partial t}\iiint_V U dV = \iint_S F ds$$

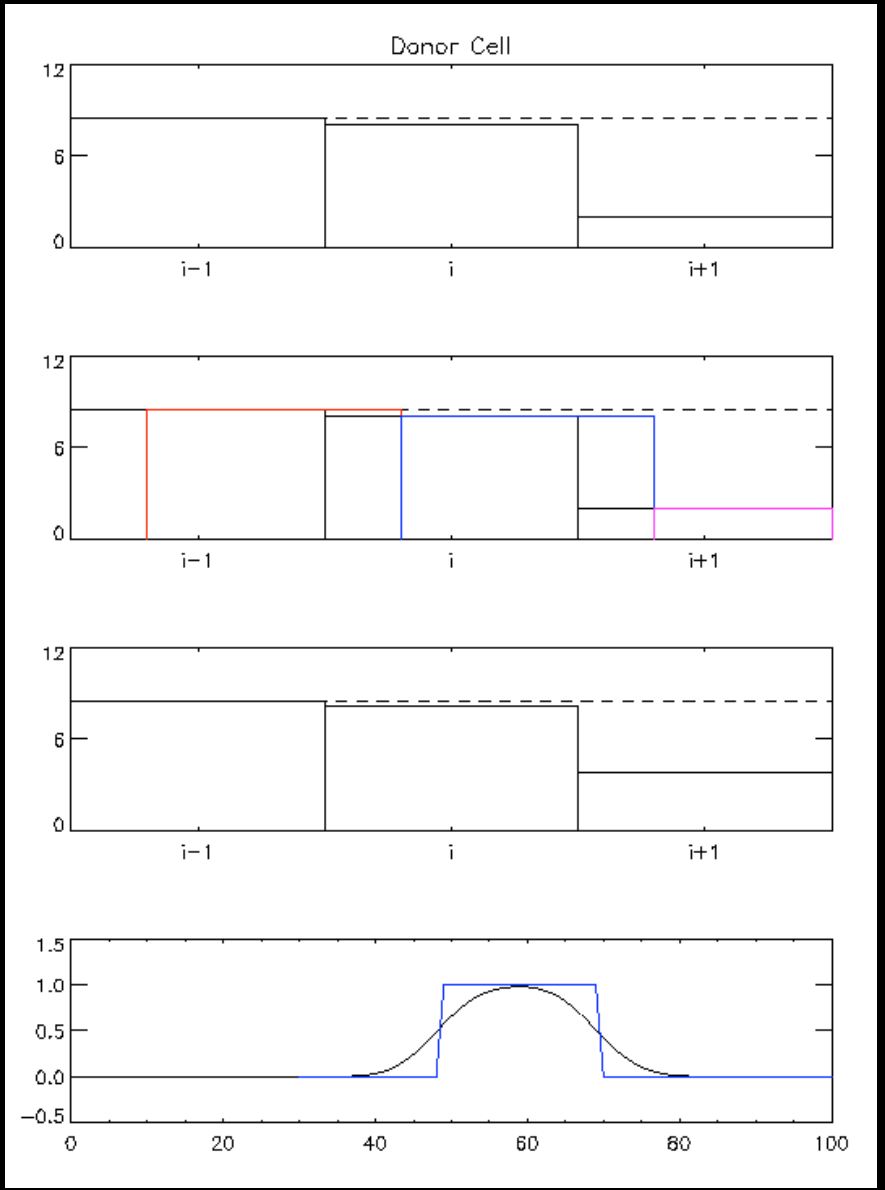
Donor Cell

- A simple first order algorithm

$$u_i^{n+1} = u_i^n + \frac{v\Delta t}{\Delta x}\left(F_{i-1/2} - F_{i+1/2}\right)$$

$$= u_i^n + \frac{v\Delta t}{\Delta x}\left(u_{i-1}^n - u_i^n\right)$$

- Maintains monotonic solution
- Linear advection problem clearly shows diffusive character



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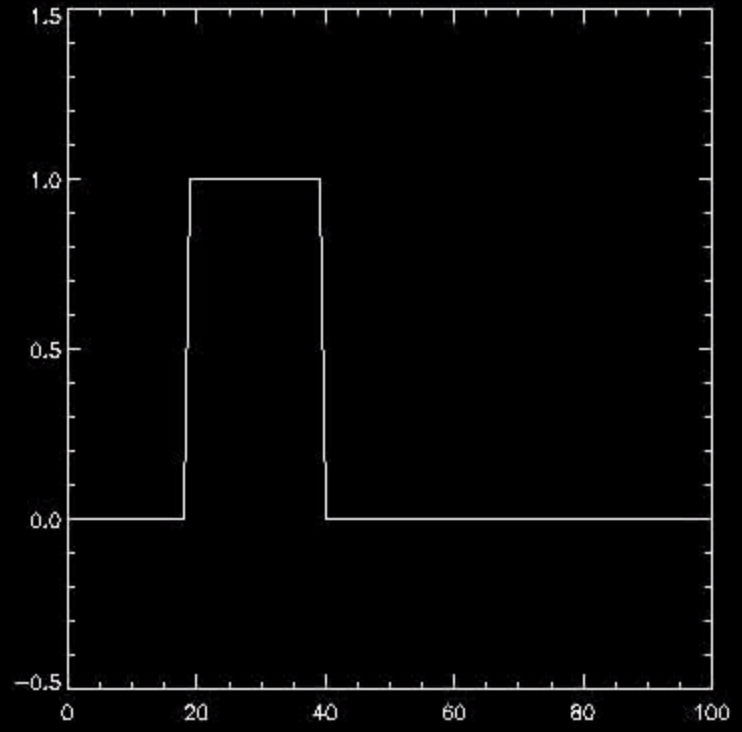
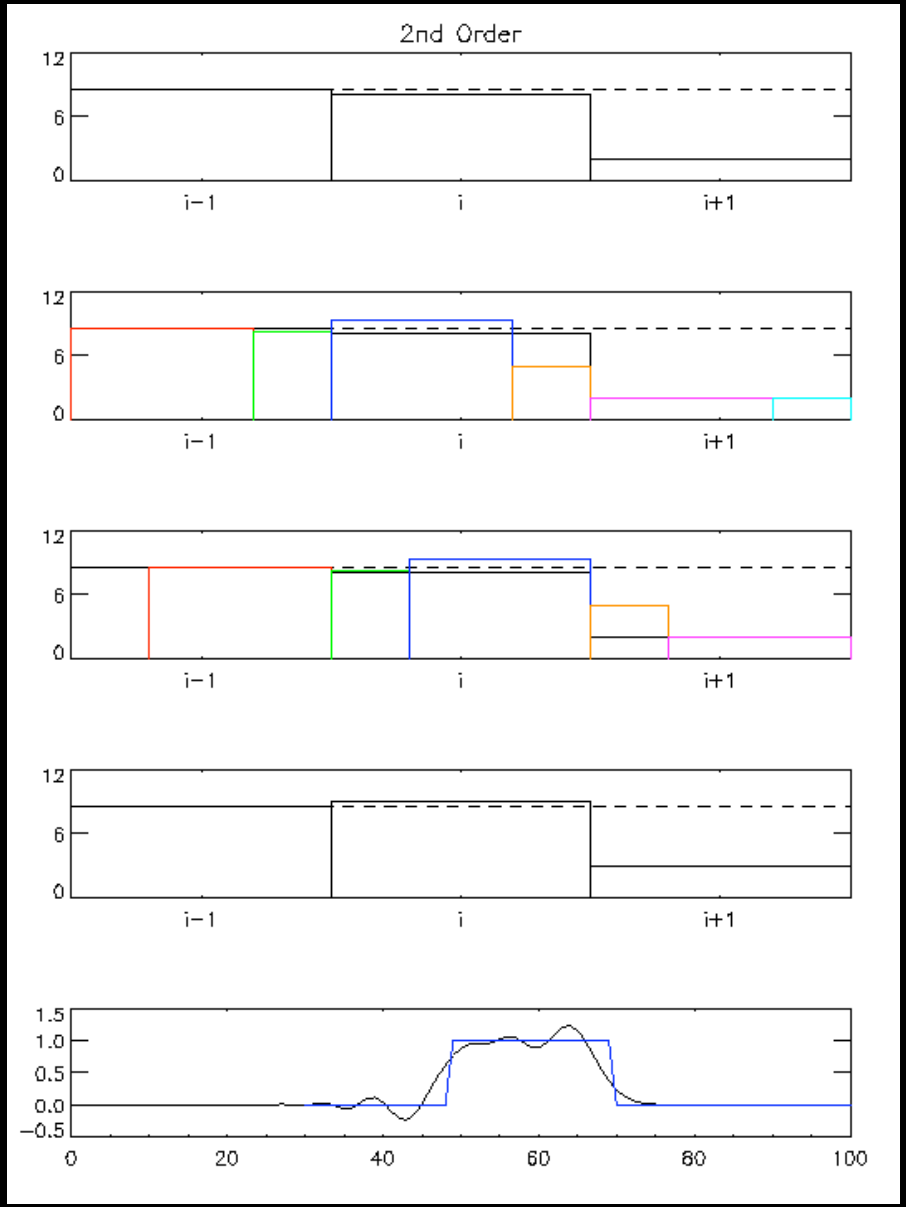


Second Order



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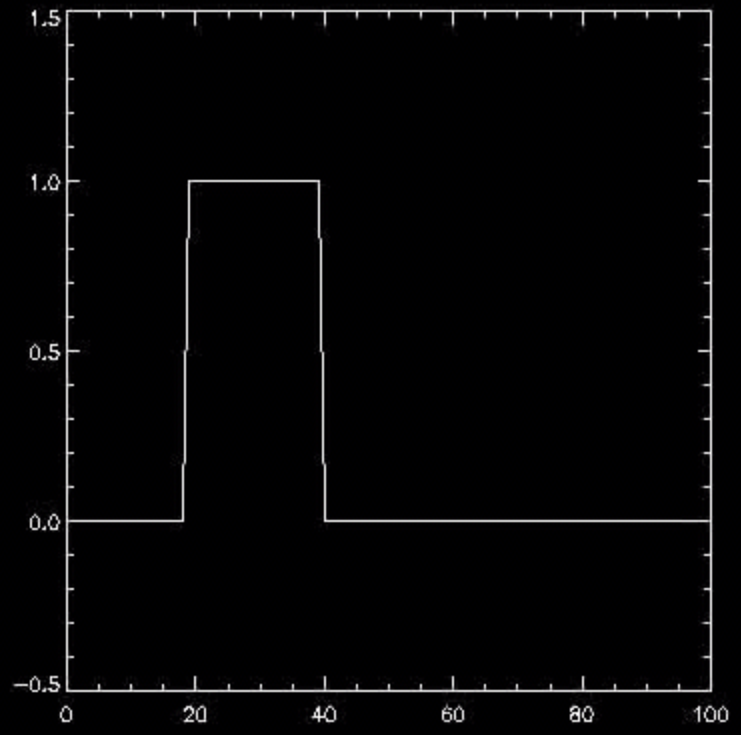
$$= u_i^n + \frac{v\Delta t}{\Delta x}\left(\frac{1}{2}\left(u_{i-1}^n - u_i^n\right) - \frac{1}{2}\left(u_i^n - u_{i+1}^n\right)\right)$$

- Does not maintain monotonic solution
- Introduces dispersion errors as seen in linear advection example

Second Order



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Partial Interface Method

- Combines low order and high order fluxes

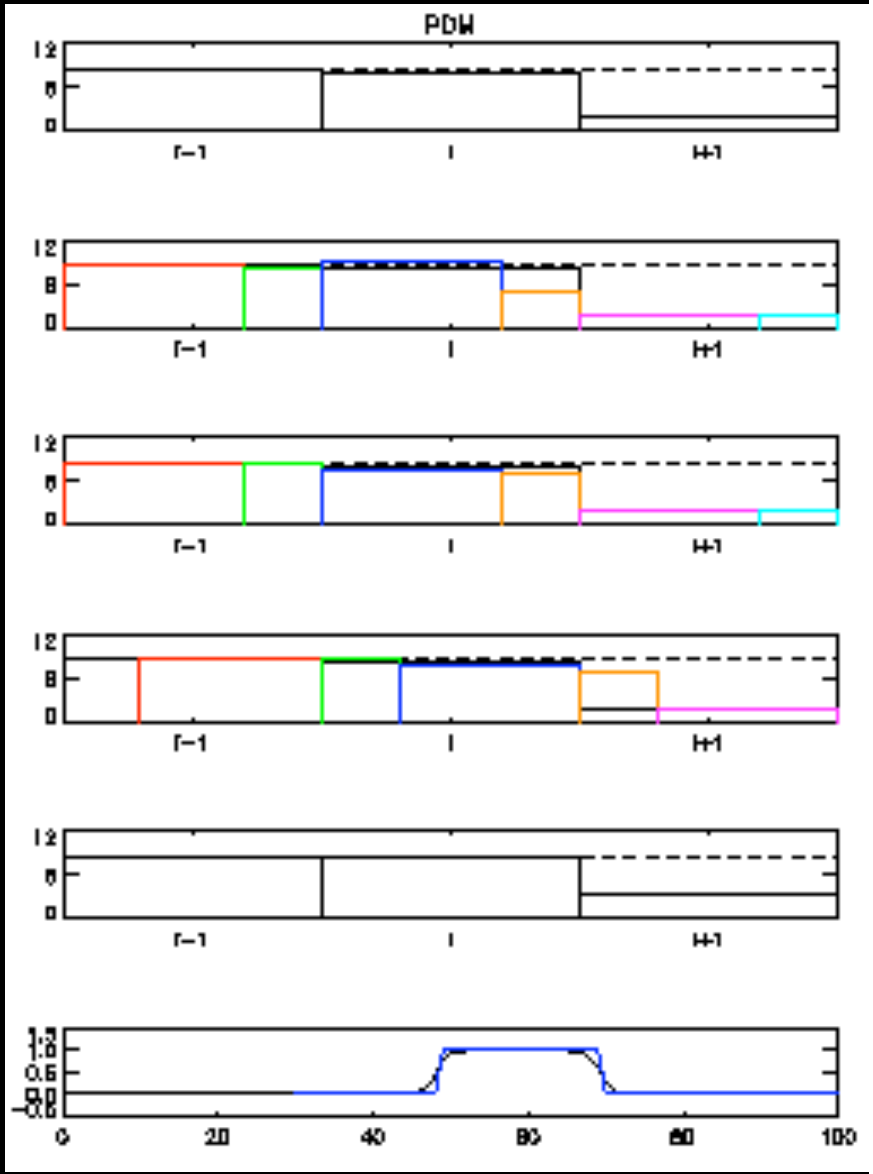
$$u_i^{n+1} = u_i^n + \frac{v\Delta t}{\Delta x}\left(F_{i-1/2} - F_{i+1/2}\right)$$

$$F_{i+1/2} = \frac{1}{2}\left(u_i^n + u_{i+1}^n\right) + \frac{1}{2} sign\left(u_{i+1}^n - u_i^n\right) *$$

$$* \max\left(0, \left|u_{i+1}^n - u_i^n\right| - Bs_i \left|u_i^n - u_{i-1}^n\right|\right)$$

$$s_i = \frac{1}{2}\left|sign\left(u_{i+1}^n - u_i^n\right) + sign\left(u_i^n - u_{i-1}^n\right)\right|$$

- Limiter to keeps solution monotonic
- Provides nonlinear numeric resistivity and viscosity



Partial Interface Method

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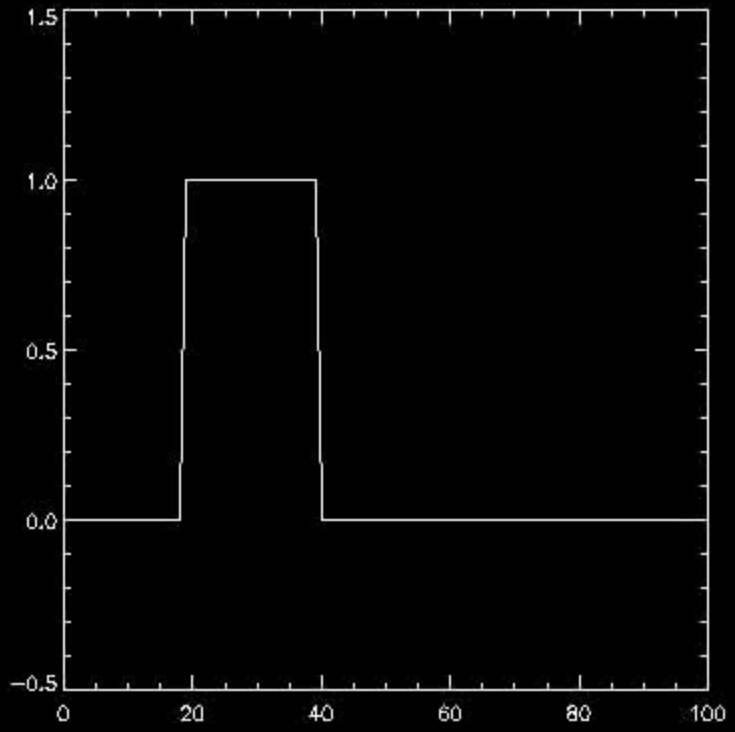


Treatment of the Magnetic Field

- Various approaches can be used to satisify the constraint that $\nabla \bullet \mathbf{B}=0$
  - Projection method

$$\nabla^2 \Psi = -\nabla \bullet \mathbf{B}$$

$$\mathbf{B}' = \mathbf{B} + \nabla \Psi$$

  - $\nabla \bullet \mathbf{B}$ convection
    - Modify the MHD equations so that $\nabla \bullet \mathbf{B}$ convects through the system

$$\frac{d(\nabla \bullet \mathbf{B})}{dt} = 0$$

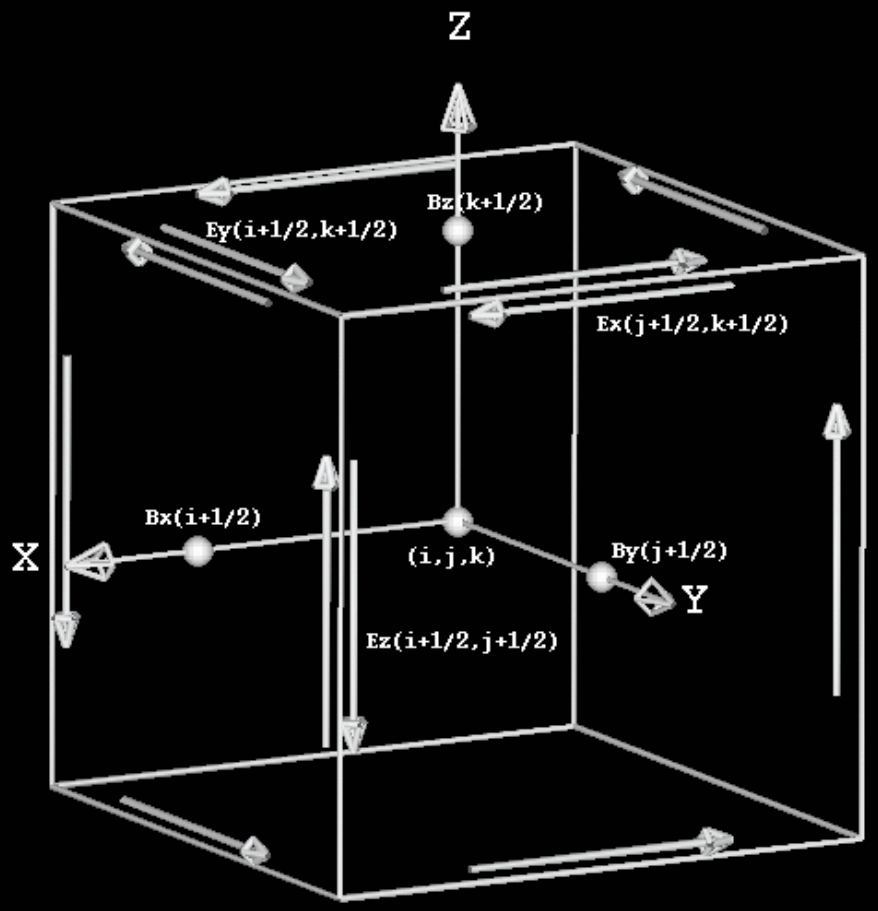
  - Use a magnetic flux conservative scheme that keeps $\nabla \bullet \mathbf{B}=0$

Magnetic Flux Conservative Scheme

- Magnetic field placed on center of cell faces
- Electric field is placed at center of cell edges so that

$$\frac{\partial}{\partial t}\left(B_x\right)_{i+\frac{1}{2},j,k} = \left[\left(E_y\right)_{i+\frac{1}{2},j,k+\frac{1}{2}} - \left(E_y\right)_{i+\frac{1}{2},j,k-\frac{1}{2}}\right]/\Delta z - \left[\left(E_z\right)_{i+\frac{1}{2},j+\frac{1}{2},k} - \left(E_y\right)_{i-\frac{1}{2},j+\frac{1}{2},k}\right]/\Delta y$$

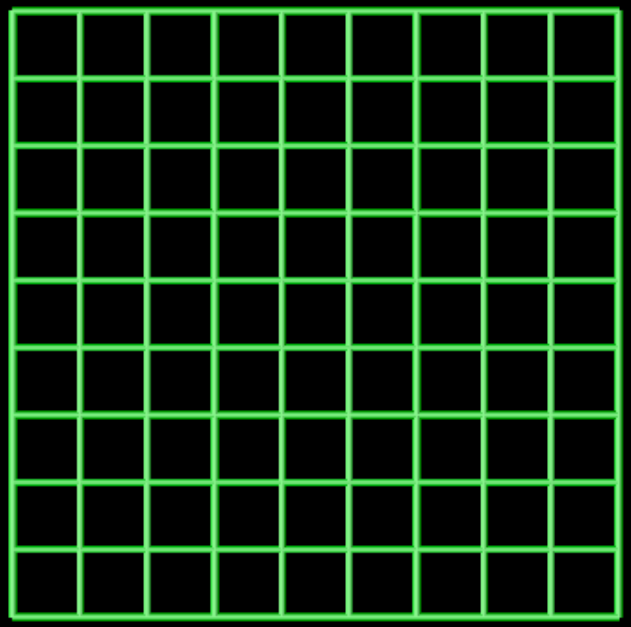
- Cancellation occurs when field components of all six faces are summed up

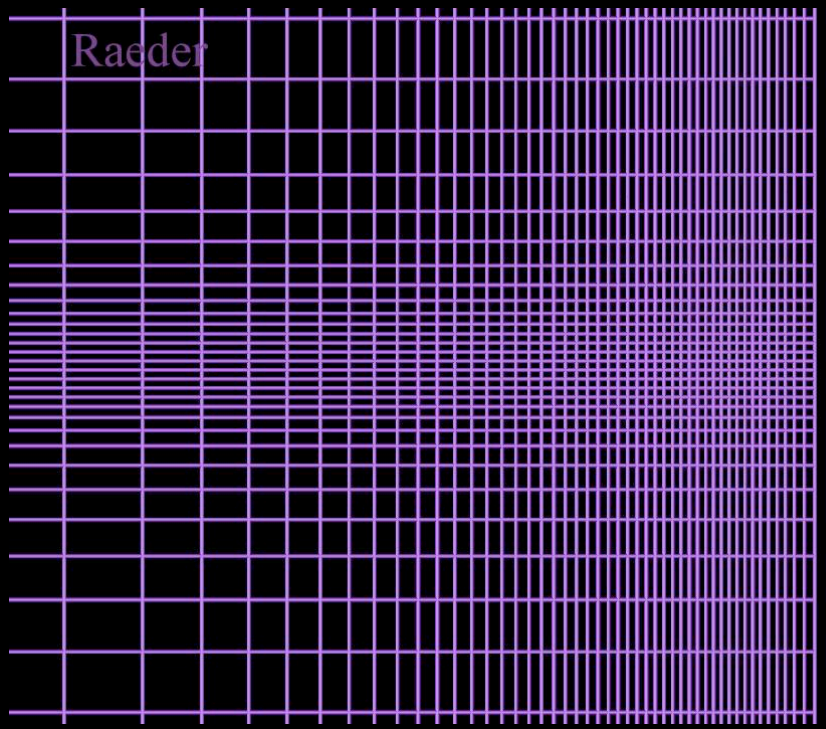


Computation Grids

- Simulation boundaries should be in supermagnetosonic flow regimes
  - $\geqslant$ 18 Re from Earth on Sunward side
  - $\geqslant$ 200 Re in tailward direction
  - $\geqslant$ 50 Re in transverse directions
- A variety of grid types exist with varying degrees of complexity
  - Uniformed Cartesian
  - Stretched Cartesian
  - Nested Cartesian
  - Regular Noncartesian
  - Irregular Noncartesian

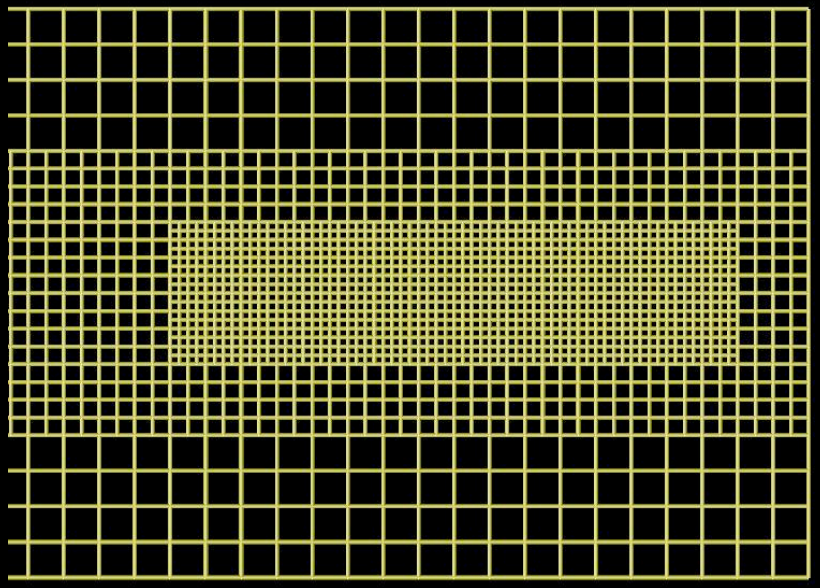
- Uniformed Cartiesian Grid
  - Low programming overhead
  - Low computing overhead
  - No memory overhead
  - Easy parallelization
  - Not very adaptable





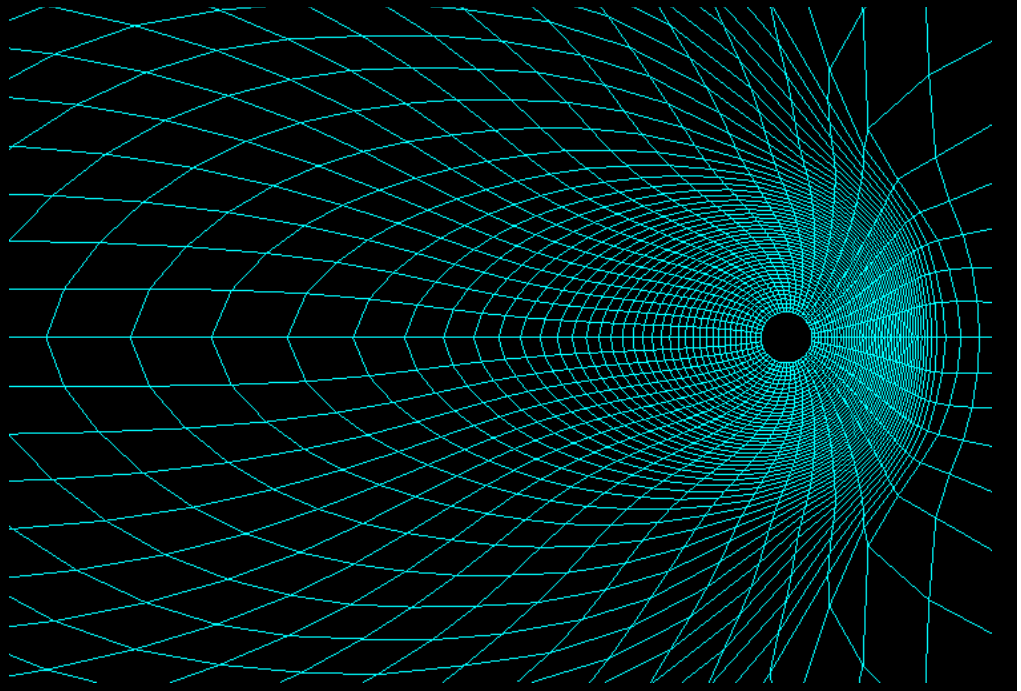
- Stretched Cartesian Grid
  - Low programming overhead
  - Low computing overhead
  - No memory overhead
  - Easy parallelization
  - Somewhat adaptable
    - Example from Raeder UCLA MHD Model

BATS-R-US

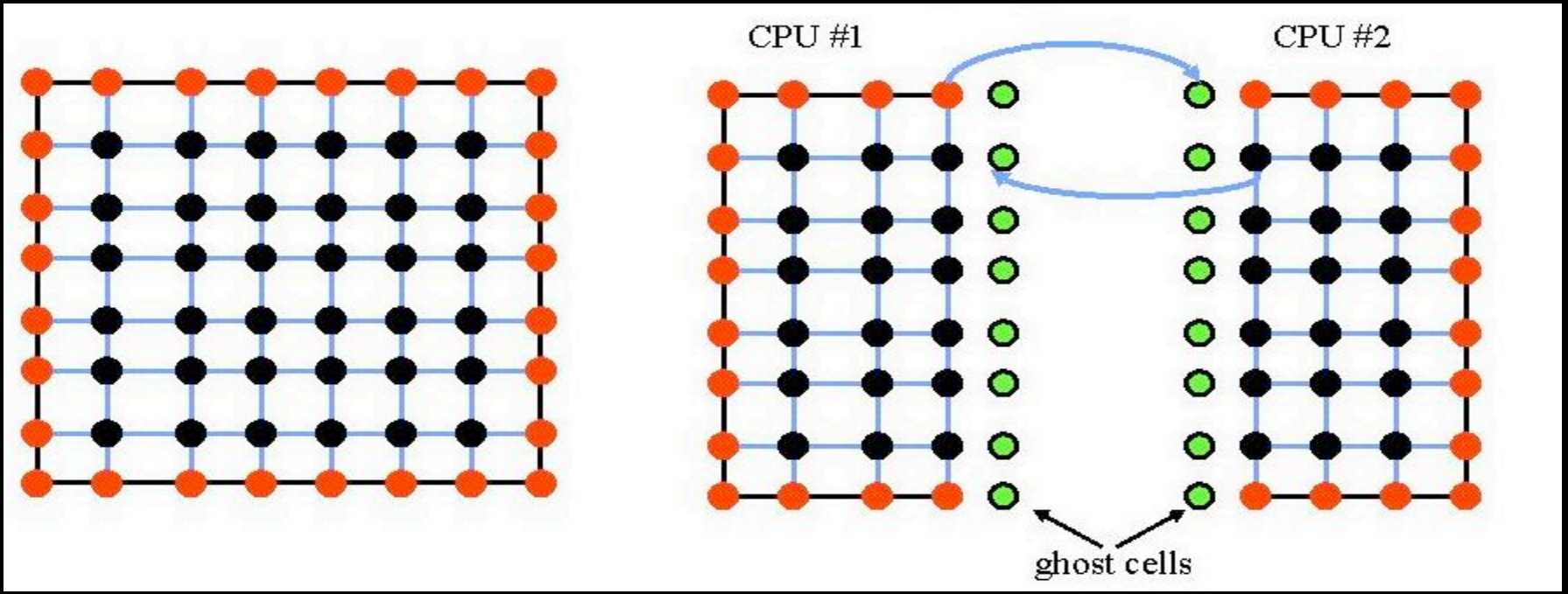


- Nested Cartesian
  - Medium/High programming overhead
  - Medium/High memory overhead
  - small computational overhead
  - difficult to parallelize
  - very (self) adaptable
    - Example from BATS-R-US

- Regular Noncartesian
  - Medium programming overhead
  - Low memory overhead
  - small computing overhead
  - parallelizes like regular cartesian grid
  - somewhat adaptable
    - Example from LFM



Domain Decomposition



- Computational Space is divided (evenly?) amongst the CPUs available to work on the problem
  - MPI used to pass boundary information between ghost cells at interfaces
  - Can also use packages like MultiBlockParti and P++

Boundary Conditions

- Upstream
  - Fixed or time dependent values for 8 plasma parameters
    - Can be idealized for derived from solar wind observations
  - Problem with B_X
    - Need to know 3D structure of solar wind because

$$\nabla \bullet \mathbf{B} = 0 \Leftrightarrow \mathbf{n} \bullet (B_{upstream} - B_{downstream}) = 0$$

    - Implies B_X=B_N cannot change if solar parameters are independent of Y and Z
    - Find **n** direction with no variation and then sweep these fronts across front boundary

Boundary Conditions II

- All other sides
  - Free flow conditions for plasma and transverse components of **B**

$$\frac{\partial \Psi}{\partial \mathbf{n}} = 0$$

  - normal component of **B** flows from [illegible] **B**=0
- Inner Boundary Condition
  - MI Coupling module
  - Hard wall boundary condition for normal component of velocity and density

Magnetosphere-Ionosphere Coupling

- Inner boundary of MHD domain is placed between 2-4 R_E from the Earth
  - High Alfven speeds in this region would impose strong limitations on global step size
  - Physical reasonable since MHD not the correct description of the physics occuring within this region
  - Covers the high latitude region of the ionosphere (45°-90°)
- Parameters in MHD region are mapped along static dipole field lines into the ionosphere
- Field aligned currents (FACs) and precipitation parameters are used to solve for ionospheric potential which is mapped back to inner boundary as boundary condition for flow

$$\mathbf{v} = \frac{(-\nabla\Phi)\times\mathbf{B}}{B^2}$$

Ionosphere Model

- 2D Electrostatic Model
  - $\nabla\cdot(\Sigma_P+\Sigma_H)\nabla\Phi=J_{\parallel}$
  - Φ=0 at low latitude boundary of ionosphere
- Conductivity Models
  - Solar EUV ionization
    - Creates day/night and winter/summer asymmetries

$$\Sigma_p = 0.5F_{10.7}^{2/3}(\cos\chi)^{2/3} \quad \forall \quad \chi \le 65^o$$

$$\Sigma_H = 1.8F_{10.7}^{1/2}\cos\chi \quad \forall \quad \chi \le 65^o$$

  - Auroral Precipitation
    - Empirical determination of energetic electron precipitation

Auroral Precipitation Model

- Empirical relationships are used to convert MHD parameters into an average energy and flux of the precipitating electrons
  - Initial flux and energy

$$\varepsilon_o = \alpha c_s^2 \qquad \phi_o = \beta \rho \varepsilon_o^{1/2}$$

  - Parallel Potential drops (Knight relationship)

$$\varepsilon_{\parallel} = \frac{R J_{\parallel} \varepsilon_o^{1/2}}{\rho}$$

  - Effects of geomagnetic field

$$\varepsilon = \varepsilon_o + \varepsilon_{\parallel}$$

$$\phi = \phi_o \left(8 - 7 e^{\frac{-\varepsilon_{\parallel}}{7\varepsilon_o}} \right) \quad \forall \quad \varepsilon_{\parallel} > 0$$

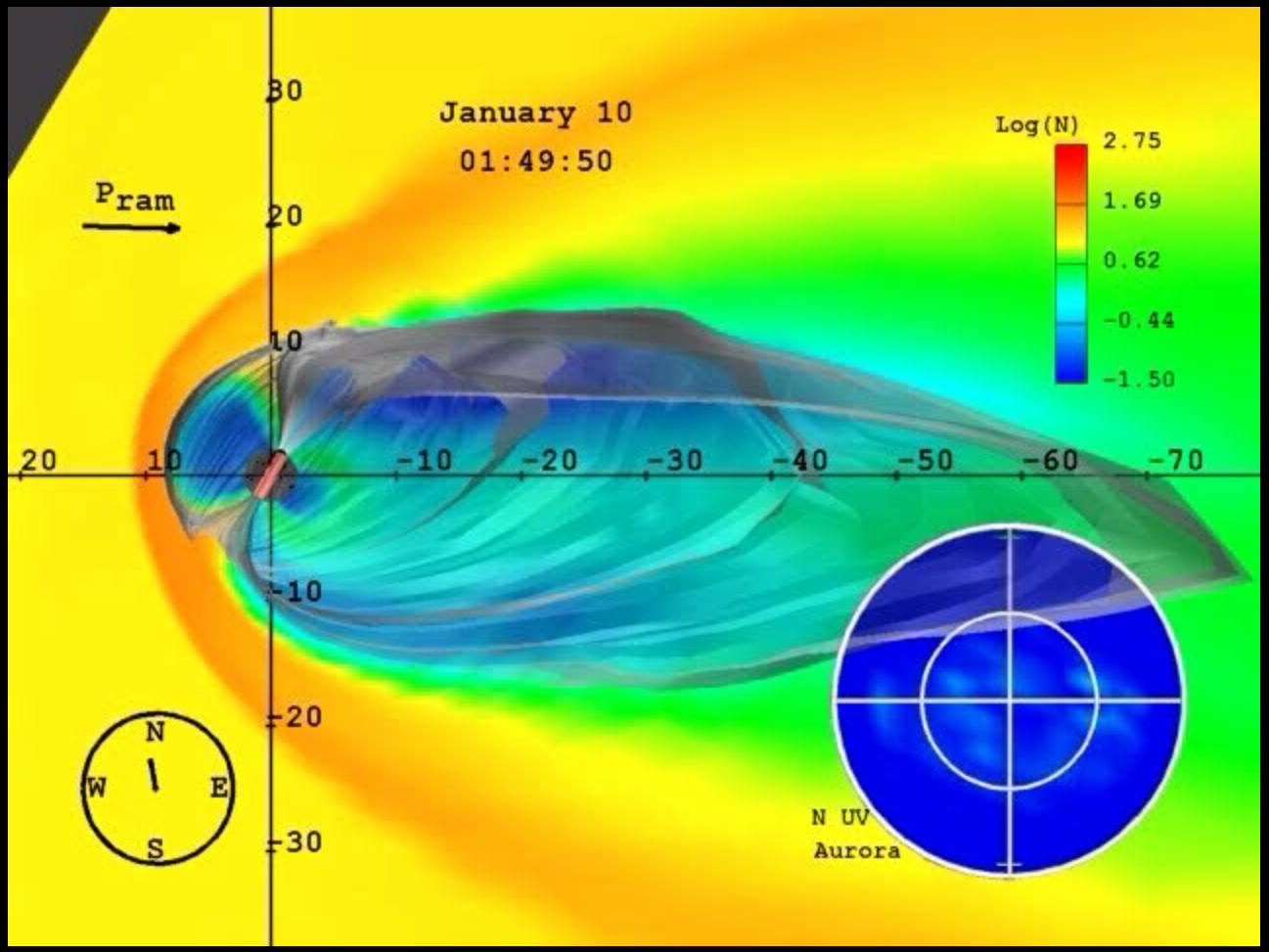
$$\phi = \phi_o e^{\frac{\varepsilon_{\parallel}}{\varepsilon_o}} \quad \forall \quad \varepsilon_{\parallel} < 0$$

  - Hall and Pederson Conductance from electron precp (Hardy)

$$\Sigma_p = \frac{5\varepsilon^{3/2}\phi^{1/2}}{\left(1 + 0.0625\varepsilon^2\right)} \qquad \Sigma_{\mathrm{H}} = 0.45\varepsilon^{0.85}\Sigma_p$$

Pretty Picture Time

- A whole lot of coding later and you get

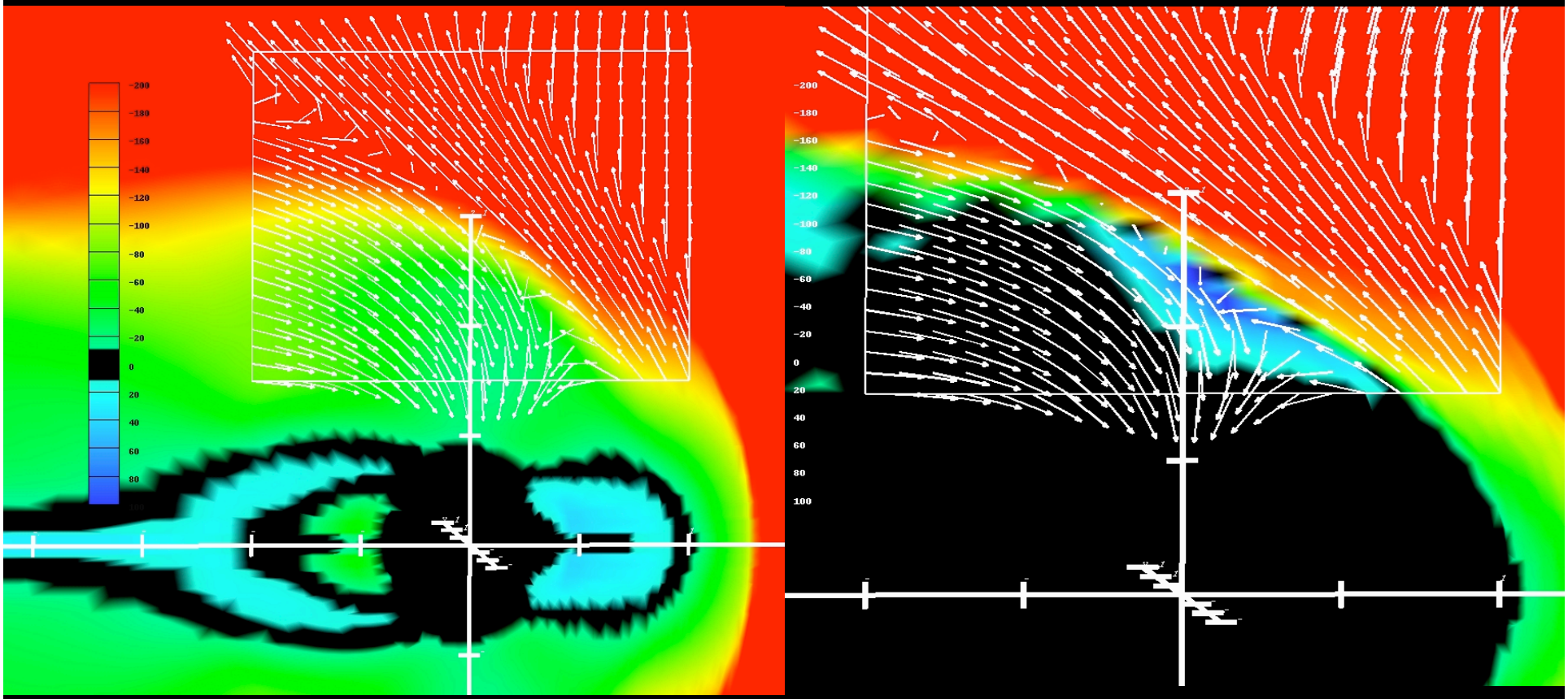


Methods of Model Validation

- Conduct studies with same conditions and different numerics
- Computation of theoretical problems with known analytic answers
  - Provides a ground truth that code is working
  - Very limited number of MHD problems
- Direct comparison with observations
  - Limited number of spacecraft observations
    - Check general characteristics with superposed epoch studies
  - Include comparison with indirect observations
  - Use metrics to quantitatively asses validity

Effect of Numerics on Magnetosphere

- Simulation for Northward IMF with constant Pedersen Conductance
  - Background color velocity with white magnetic field vectors

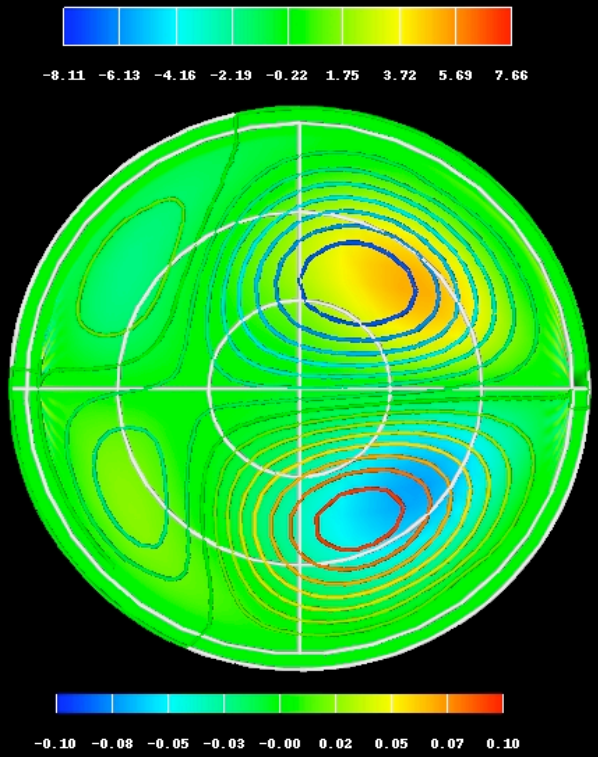


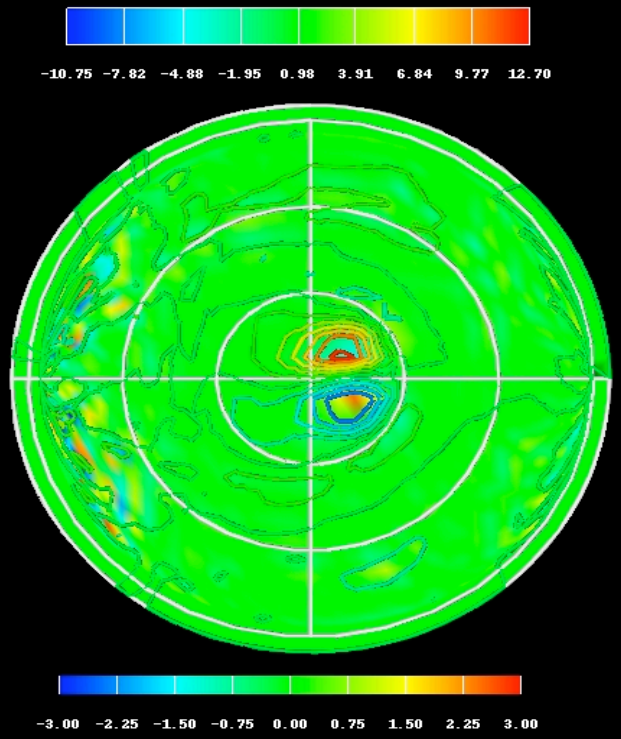
- High Numerical Diffusion
  - 8th Order
  - No TVD Scheme

- Low Numerical Diffusion
  - 8th Order
  - High TVD Scheme

Effect of Numerics on Ionosphere

- Simulation for Northward IMF with constant Pedersen Conductance
  - Background color FAC strength with potential contours overlaid





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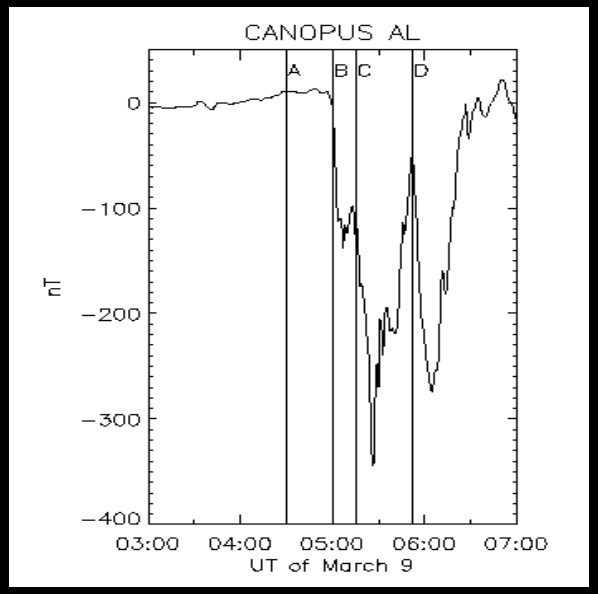
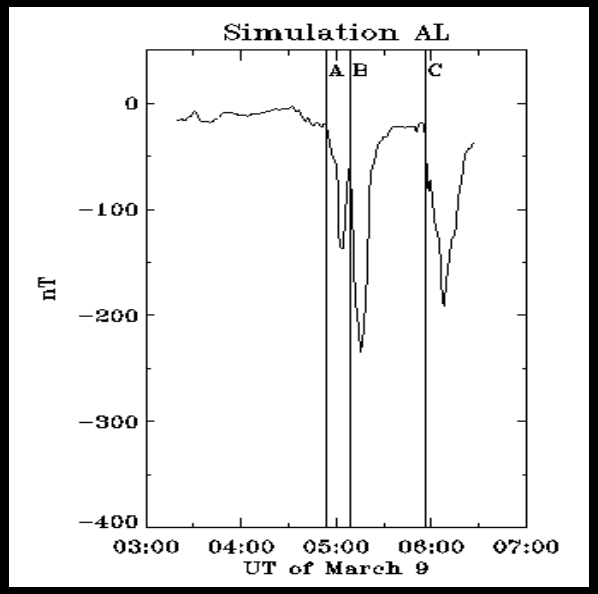
  - High TVD Scheme

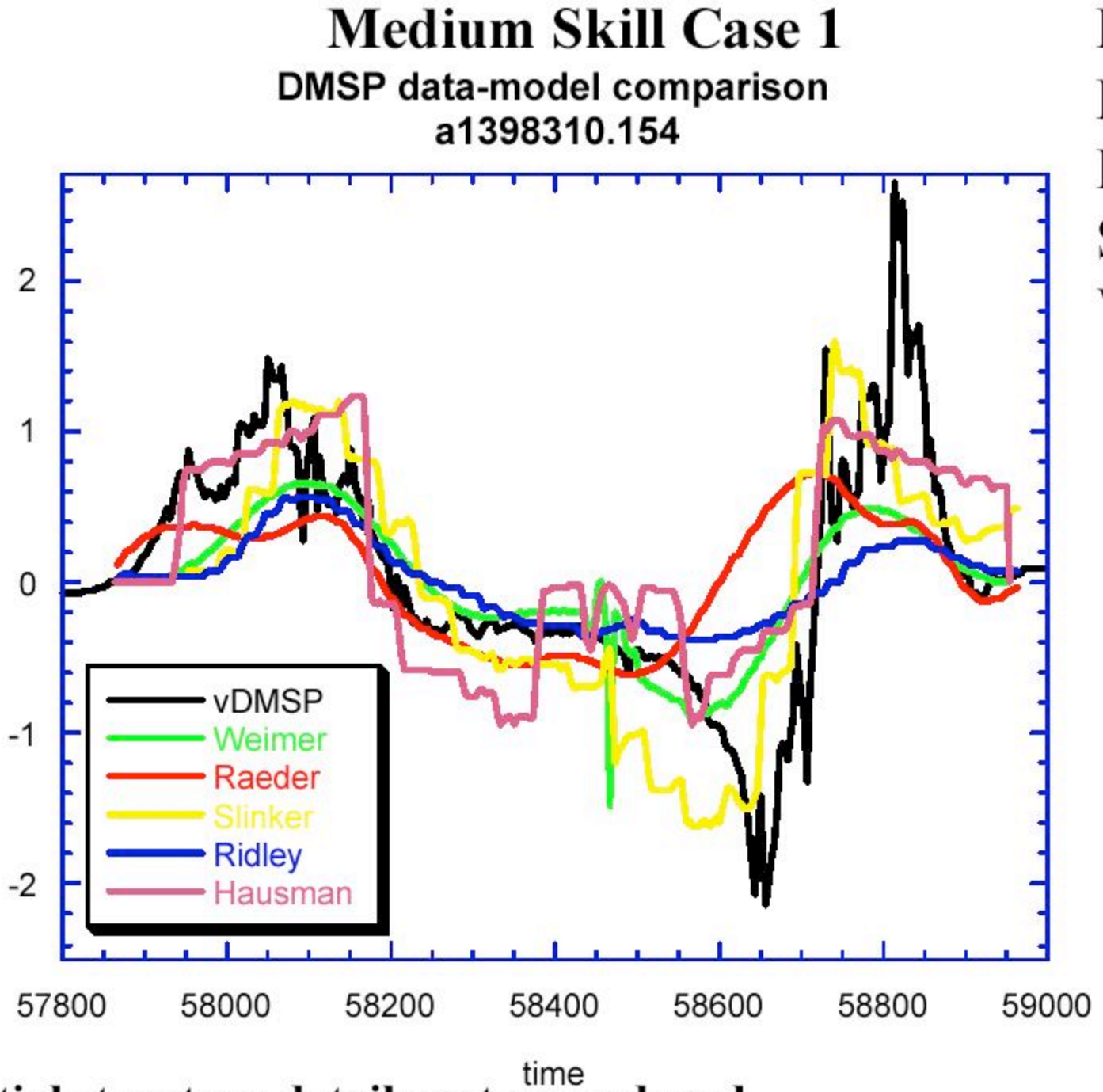
Energy loading and unloading





- Both data and simulation show onset, intensification, recovery, and second onset
- Simulated onset is early, but intervals between intensification and second onset are consistent
- Simulated CL recovers faster than observations

Medium Skill Case 1



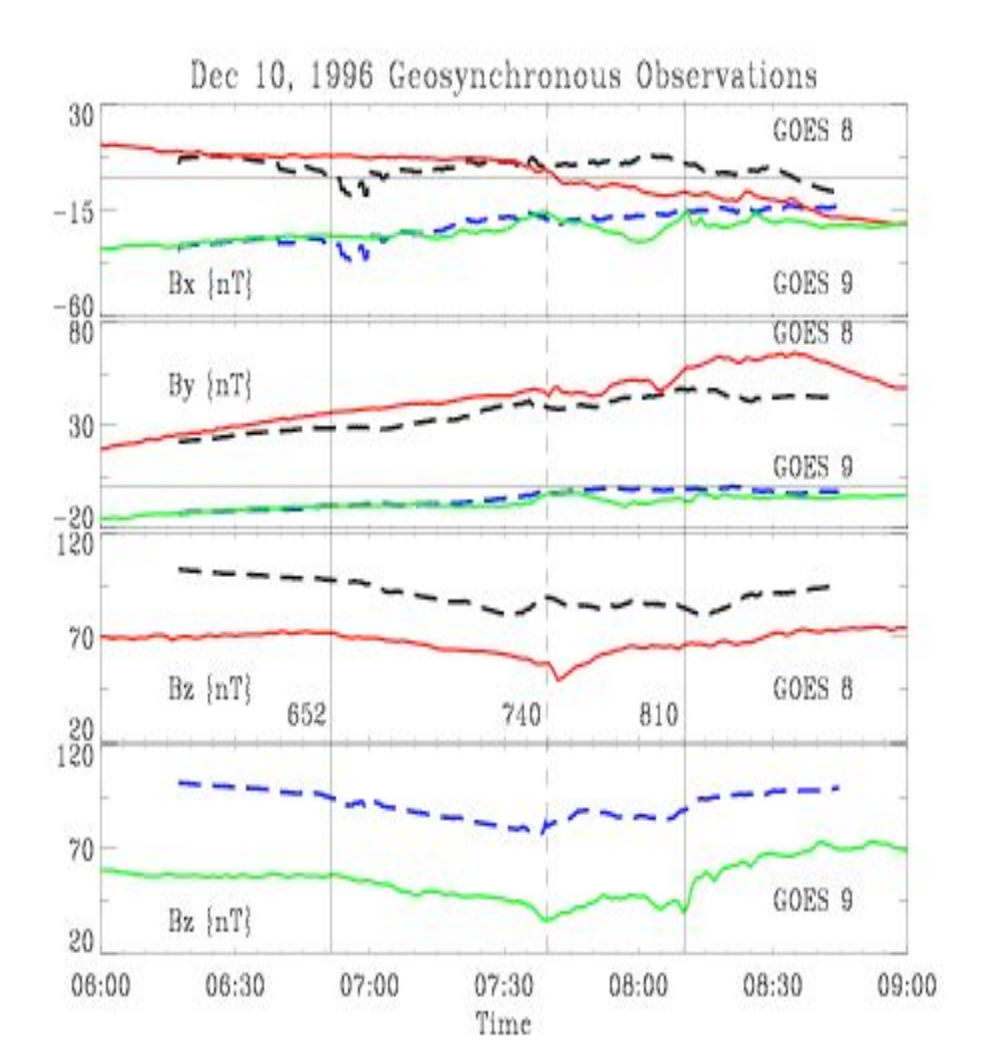
Raeder: 0.27
Ridley: 0.32
Rice: 0.32
Slinker: 0.23
Weimer: 0.43

Spatial structure details not reproduced

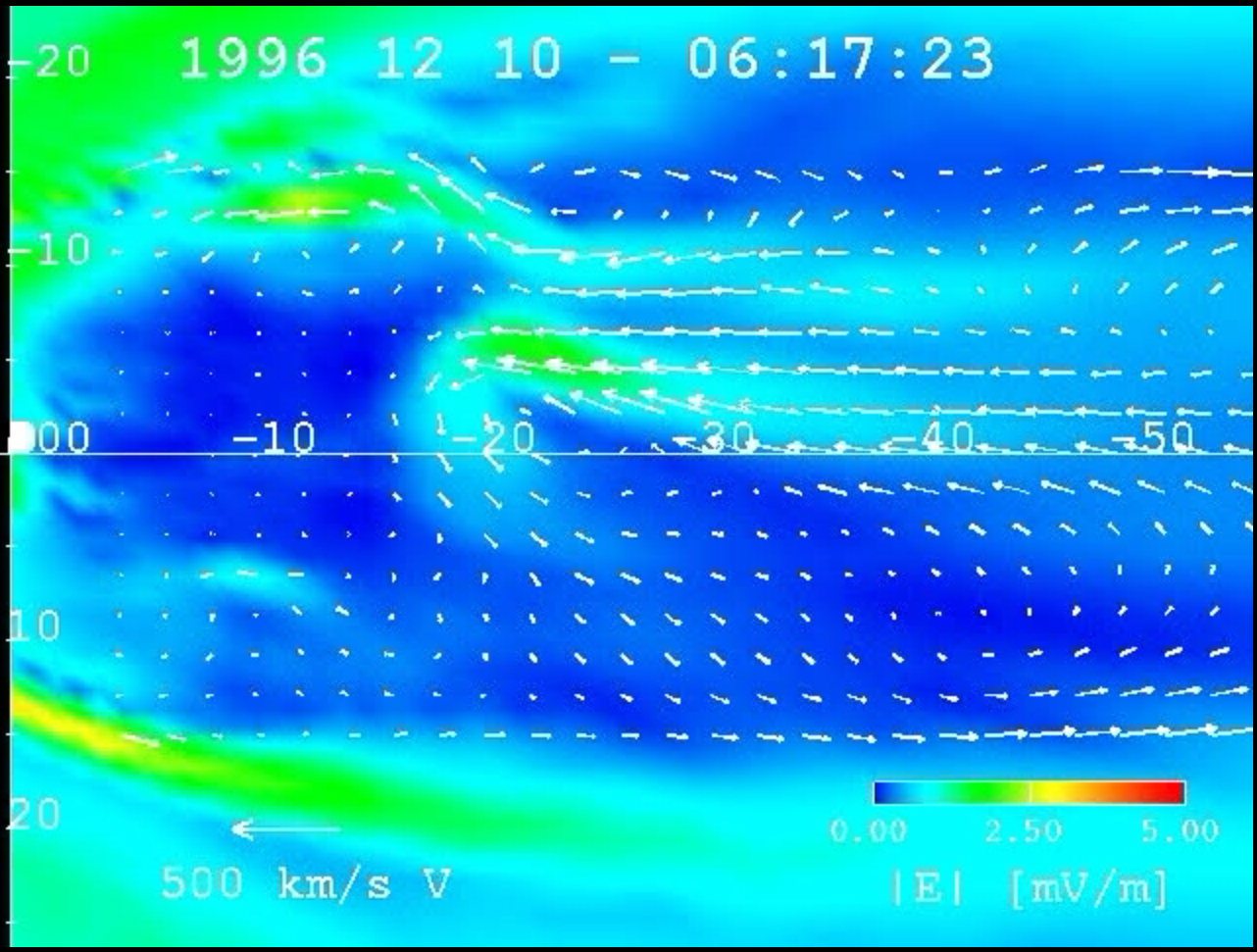
More detail (Rice, Slinker) does not necessarily lead to higher skill, although it is desirable

Comparison with geostationary observations

- Excellent agreement for all three components of B
- Despite global B_Z offset dipolarizations of similar size are seen in simulation results for both GOES 8 & 9
  - May imply limited role for ring current in substorms

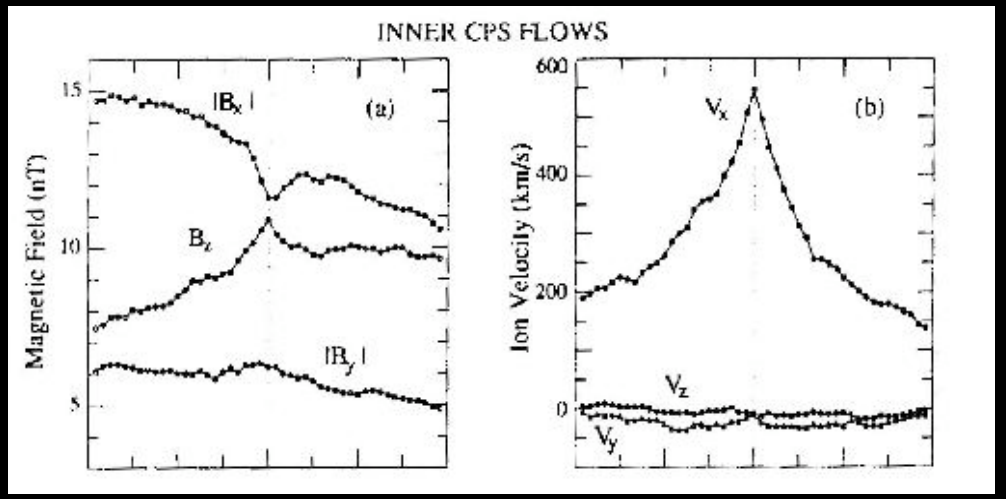


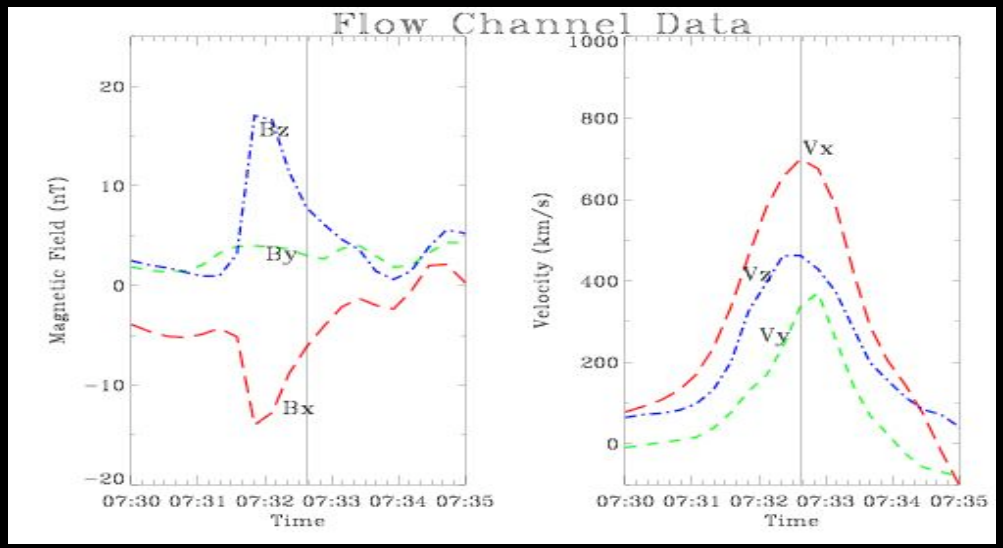
Flow Channels

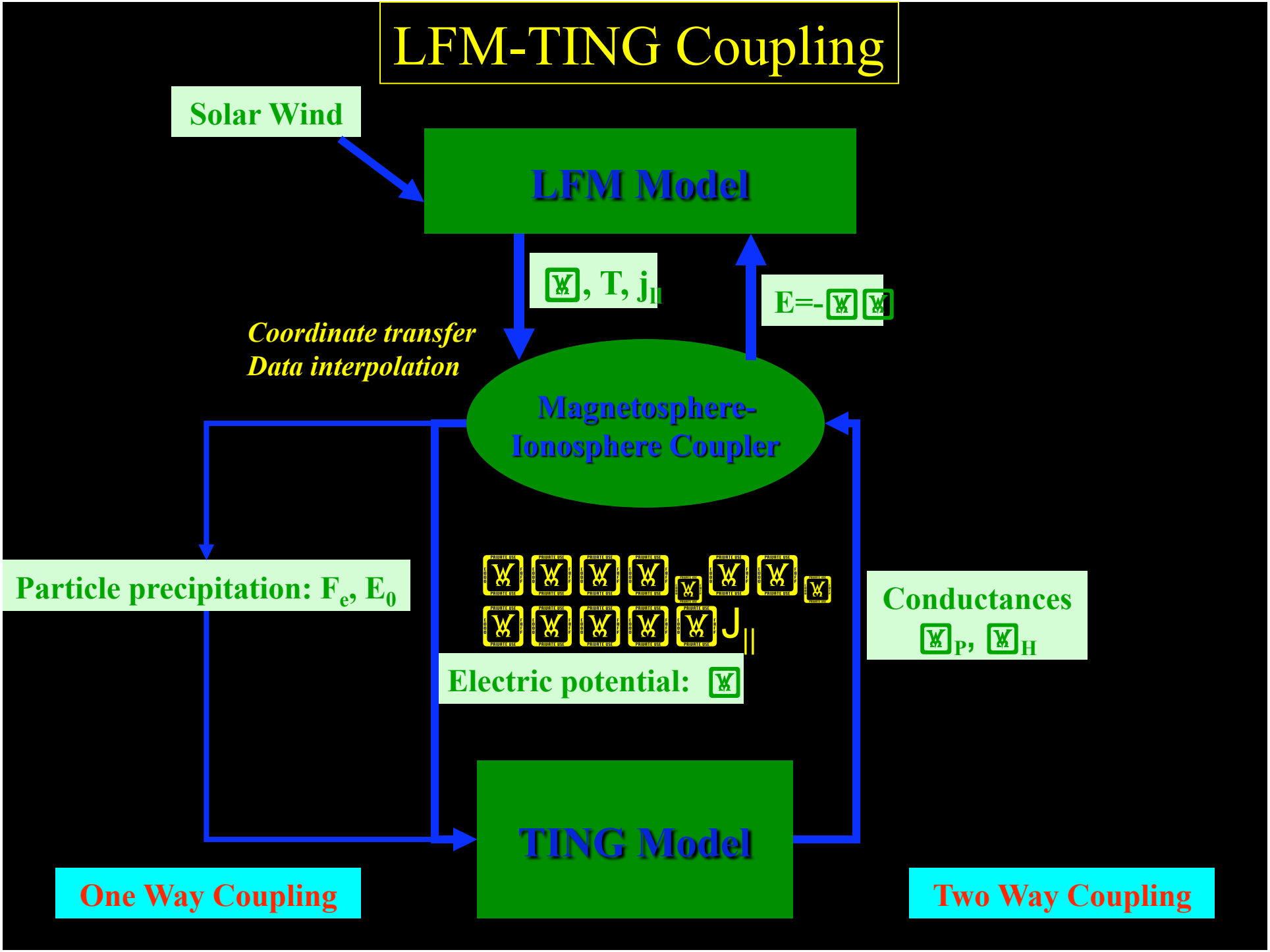


Comparison between Flow channels and BBFs

- Flow channels have properties similar to BBF results reported by Angelopolous
- FWHM of V_X profile and magnitude comparable BBF properties
- Use code to determine if they result from localized reconnection or interchange instability






LFM-TING Coupling
Solar Wind
LFM Model
, T, j||
E=-
Coordinate transfer
Data interpolation
Magnetosphere-
Ionosphere Coupler
Particle precipitation: Fe, E0
J||
Electric potential:
Conductances
P, H
TING Model
One Way Coupling
Two Way Coupling

Conclusions

- Global MHD simulation of the magnetosphere under idealized solar wind conditions are proving to be a useful tool for expanding our understanding the coupled solar wind – magnetosphere – ionosphere system
- The technique is expanding into new frontiers
  - Ionospheric simulation is being replaced with more sophisticated Thermosphere-Ionosphere Global Circulation Models
  - Modeling of the inner magnetosphere is being enhanced by coupling with the Rice Convection Model

Conclusions

- LFM is highly successful global MHD simulation of the magnetosphere
  - Numerous publications and presentations
  - Its design considerations are still relevant today
- LFM is still evolving
  - Ports to new platforms and utilization of MPI
  - Ionospheric simulation is being replaced with more sophisticated Thermosphere-Ionosphere Global Circulation Models
  - Modeling of the inner magnetosphere is being enhanced by coupling with the Rice Convection Model