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A Method for Computing Accurate Daily Atmospheric Density Values from Satellite Drag Data

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A method has been developed for computing accurate daily density values based on satellite drag data. A differential orbit correction program using special perturbations orbit integration is used to fit radar and optical observational data to obtain the standard 6-element state vector plus a ballistic coefficient (B). The atmospheric density model used in the integration is a modified Jacchia 1970 model that was developed for incorporation into the USAF High Accuracy Satellite Drag Model (HASDM). Energy dissipation rates (EDR) are computed over the observation span (3 to 8 days) using the modified Jacchia model density values and the fitted B value. Overlapping EDR values are obtained by moving the start of each succeeding orbit fit by one day. These overlapping fits are then used to compute accurate daily EDR values. Daily temperature and density values are then computed from the observed EDR values using the "true" 30-year B value of each satellite plus using partial derivatives of temperature and EDR changes computed by the HASDM density model. The daily temperature values were validated by computing the daily values for satellites during year 2001, and comparing the results obtained using the HASDM Dynamic Calibration Atmosphere (DCA) program. The comparison is excellent between the current method described herein and the HASDM DCA method. Finally, the daily density computation was validated by comparing historical daily density values computed for the last 30 years for over 25 satellites. The accuracy of the density values was determined from comparisons of geographically overlapping perigee location data, with over 8500 pairs of density values used in the comparisons. The density errors are less than 4% overall, with errors on the order of 2% for values covering the latest solar maximum.

INTRODUCTION

Atmospheric density variations determined from satellite drag data have been the subject of numerous studies since the dawn of the space age. Previous methods have been

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developed to obtain density variations from analysis of satellite orbit decays, using either mean orbital elements, or raw observational tracking data. Numerous studies[1] have been conducted and published using the standard NORAD mean elements to obtain density variations over time. The results have led to a much better understanding of the various density changes that occur. However, the accuracy of the density changes has been limited because of the inherent inaccuracies of the mean orbital elements, which were obtained using a general orbit perturbation theory[2]. Also leading to the inaccuracies of the mean orbit elements is the unknown observation span used to fit the elements. The unknown observation span results in obtaining, not daily decay rates, but decay rates averaged over many days. Other methods to analyze orbit decay rates have used special orbit perturbations to fit the satellite observations. However, using special perturbations requires availability of very accurate radar and optical observations to obtain accurate orbit fits, and then the challenge is to obtain daily density variations over the averaged orbit fit span. The method described herein uses special perturbations orbit determination employing a daily orbital energy dissipation rate method to obtain very accurate daily variations. Radar and optical observations from the Air Force's Space Surveillance Network (SSN) of worldwide sensors are used in the orbit fitting to obtain precise orbital elements. A key factor in the method is obtaining true ballistic coefficients to be able to compute the daily observed density variations from the observed daily energy dissipation rates. Validation is another key issue in the development of any new method. This paper describes the validation of the method described herein using many thousands of computed daily density values obtained on more than 25 satellites.

DIFFERENTIAL ORBIT CORRECTIONS

A differential orbit correction program is used to fit SSN observations to obtain the standard 6 Keplerian elements plus the ballistic coefficient (B). An important part of obtaining accurate orbit fits is the determination of the observation span for each fit. If the time span is too short, the observation may still be accurately fit, in a least squares sense, but the B values will be ill determined. A minimum of 10 accurate radar observations, spread throughout the fit span, are required to obtain an accurate orbit fit, while the great majority of the fits will have at least 30 to 60 observations per fit. An example of this can be seen from analysis of orbit fits for satellite 00060, Explorer 8, with a perigee height of 400 km. Figure 1 shows the B values determined using observation spans of 1 to 4 days during 1996 when over 125 observations per day from over 10 different accurate radars were obtained on this satellite. All the orbit fits were excellent in fitting the observations. However, the B values for the 1-day span have huge variations, and are unusable. These variations cannot be attributed to real density variations since the solar flux values are mainly constant during most of the fit span time period. Even the 2-day span fits are not accurate enough to use. The selection of the correct observation span for the orbit fits is crucial in obtaining accurate B values.

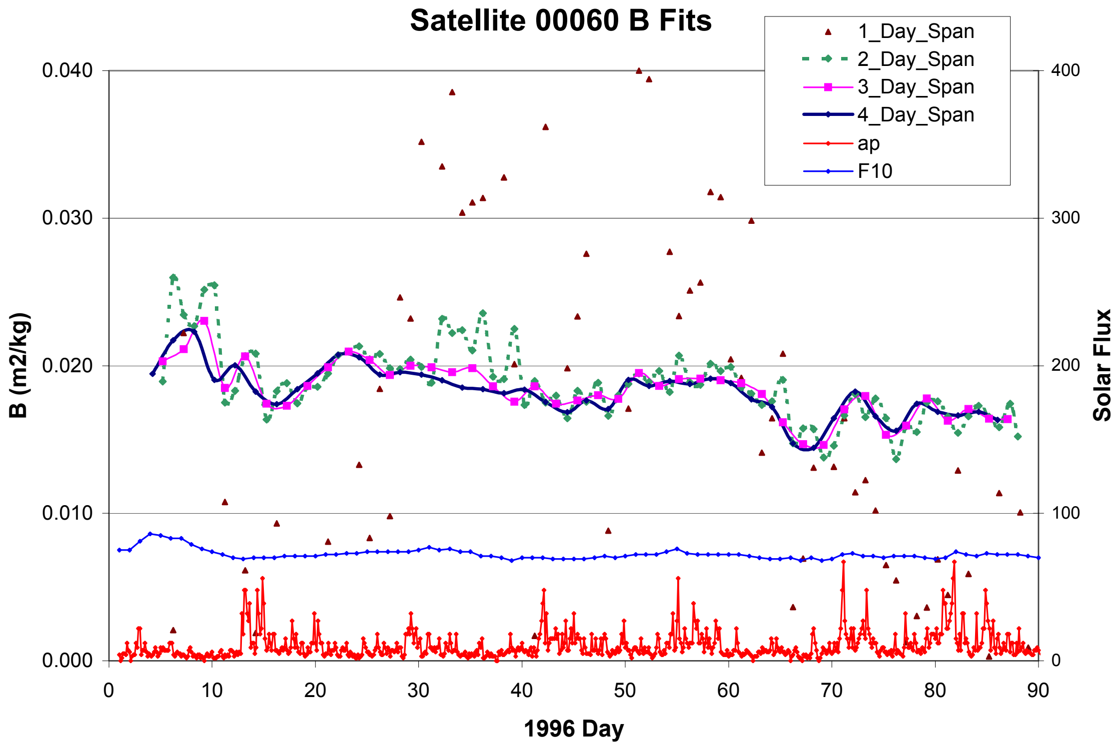

Figure 1. Ballistic coefficient values obtained for satellite 00060 with perigee height of 400 km during 1996 using different orbit fit observation spans.

The differential correction orbit fits are obtained using a weighted least squares differential orbit correction program that uses special perturbations orbit integration.

The geopotential selected for use in the differential orbit corrections was the EGM96[3] model truncated to a 48×48 field. The basis for this selection is shown Table 1 below. The table was developed by computing differential orbit corrections for satellite 00060 (Explorer 8) every 3 days from 1996 through 1999. The orbit corrections were obtained for the full 70×70 EGM96 geopotential field, and then for truncated versions from 36×36 to 48×48. The ballistic coefficient errors were then computed as the difference of the B values from each truncated field verses the B values using the reference 70×70 field fits. The observation spans and number of observations for each differential orbit correction were verified to be identical when comparing the fits for the different truncated fields. This meant that all the difference in the B value was due solely to the truncation of the geopotential. The table shows that when using a 36×36 field the maximum B error can be as large as 13.6% for this orbit. The standard deviation of B is 2.5% for the 36×36 field. These values are much too large when attempting to obtain density values accurate to within 2-3%. The use of the 48×48 field shows that the maximum errors in B are less than 4%, with a standard deviation less than 1% for this orbit with a perigee height close to 400 km. These are acceptable values for the B errors to obtain the desired density accuracy.

Model:	36x36	41x41	48x48
STD	**2.5 %**	**1.2 %**	**0.8 %**
MAX	**13.6 %**	**7.7 %**	**3.6 %**

Table 1. Ballistic coefficient percent errors (standard deviation STD and maximum deviation MAX) from orbit fits of satellite 00060, 400 km perigee height, over the period 1996 through 1999. EGM96 model with truncated terms of 36×36, 41×41, and 48×48 used.

The special perturbation integration also includes third-body gravitational effects of the sun and moon, solar radiation pressure, and accelerations due to atmospheric drag. The atmospheric density model used in the integration is a modified Jacchia[4] 1970 model that was developed for incorporation into the Air Force's High Accuracy Satellite Drag Model (HASDM) program[5]. The modified Jacchia 1970 model uses the same Jacchia equations to compute the density but also incorporates additional equations to compute new temperature and density partial derivatives for improved orbit fits.

A variable drag coefficient (C_D) approach was required for the processing. As reported in Bowman[6], the drag coefficient will vary with altitude based on the changing number densities of the molecular constituents of the atmosphere. Normally the C_D value is considered constant at a value of 2.2. However, during periods of low solar activity ($F_{10.7}$ < 80) the dominant atmospheric species at higher altitudes changes from oxygen to helium as low as 500 km altitude. The drag coefficient for oxygen is approximately 2.2, while for helium it approximates 2.8. Above 1500 km, when hydrogen becomes dominant during solar minimum, the C_D value is greater than 4.0. Therefore, the B value, consisting of the drag coefficient and area to mass ratio, can vary by as much as 80% over a wide range of altitudes. If the C_D value is not varied in the model based on molecular constituents then the error in C_D will be reflected in an error in B that is not due to atmospheric density.

Figure 2 shows the difference in the B coefficient from comparing a constant and variable C_D value. Several satellites were used to compute B values every 3 days for the 31-year period, using first a constant C_D value of 2.2 for all the fits, and then using a variable C_D value based on molecular species number densities. The differences in the figure show that at altitudes of 900 km the B value can be in error from the C_D change by as much as 18% during low solar activity.

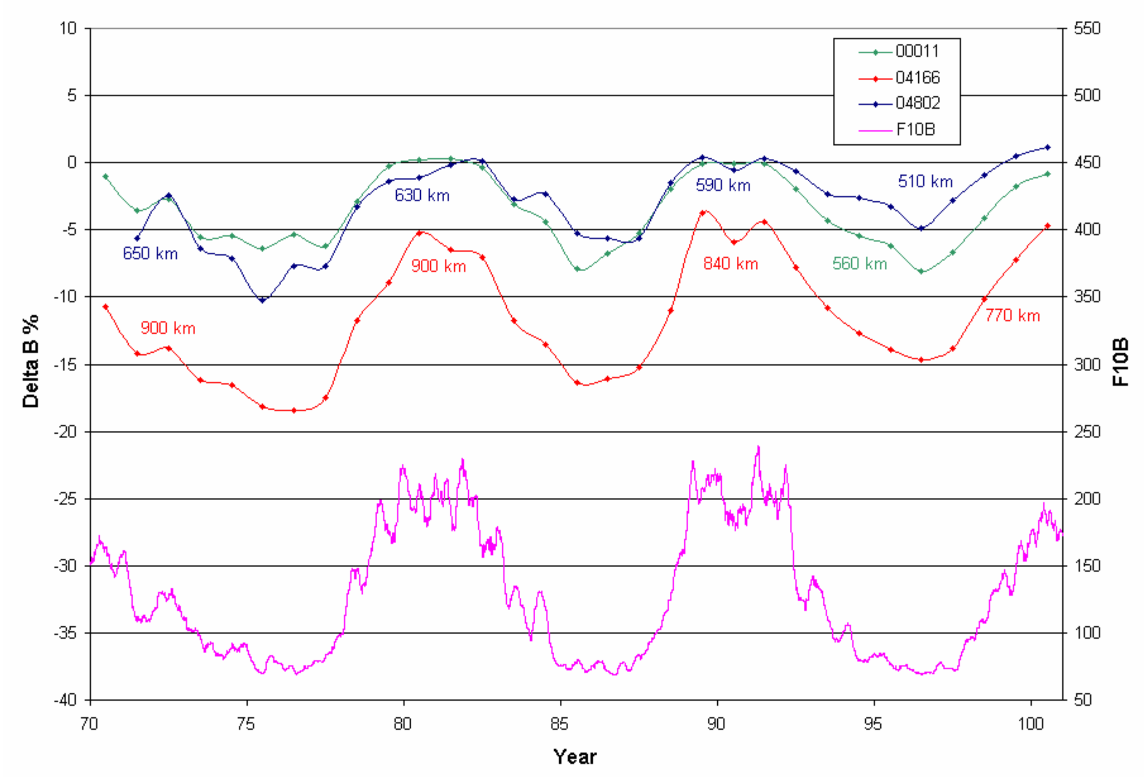

Figure 2. Ballistic coefficient, B, percent differences from using a variable C_D minus constant C_D as a function of height for several different satellite orbits. The 90-day average $F_{10.7}$, F10B, is also plotted.

ENERGY DISSIPATION RATE MODEL

To compute accurate daily density values, the differential orbit correction determination program was modified to compute orbital energy dissipation rates (EDR)[7] over the observation span (typically 3-8 days) using the modified Jacchia model density values and the fitted B value. Overlapping EDR values were obtained by moving the observation span by one day and computing another orbit fit with the same observation span length. These overlapping fits were then used to compute accurate daily EDR values using a weighting method that favors EDR values obtained from the middle of the observation span as opposed to those at the ends of the observation span. These daily EDR values were then used to compute observed daily temperature values.

Daily Energy Dissipation Rate Computations

The energy dissipation rate is defined as

$$\bar{\dot{\varepsilon}} = \frac{B}{2\Delta t}\int_0^{\Delta t} \rho V_{REL} (\vec{V}_{REL} \bullet \vec{V}_{SAT}) dt \quad (1)$$

where

B = satellite ballistic coefficient from orbit fit (m^2/kg)

Δt = time span over which EDR is computed (sec)

ρ = atmospheric model density (kg/m^3)

$\vec{V}_{REL} = \vec{V}_{SAT} - \vec{\omega} \times \vec{r}_{SAT}$ the relative velocity between the satellite and rotating atm (m/s)

$\vec{V}_{SAT}$ = satellite velocity in inertial space (m/s)

for simplicity set $\quad f(v) = \left|\vec{V}_{REL}\right|(\vec{V}_{REL} \bullet \vec{V}_{SAT})$ (2)

From the beginning of the first day following the first observation to the end of the last day following the last observation the EDR and density partials are computed daily as follows:

$$\bar{\dot{\varepsilon}}_i = \frac{B}{2 \times 86400} \sum_{j=0}^{M} \rho_{ij} f(v_{ij}) dt \quad , \quad M = 86400/dt \,, \quad i = \text{day } 1,2,3,... \tag{3}$$

where
$dt = 1$ sec for orbit eccentricity > 0.01
$dt = 6$ sec for orbit eccentricity ≤ 0.01
ρ_{ij} = density for j ephemeris point of i day ,
v_{ij} = velocity for j ephemeris point of i day

EDR values are computed for each day throughout the observation span. The sums run from the beginning to the end of each day. The B value is the orbit corrected value that is constant over the entire orbit fit span. At each time step *dt* the density and velocities are obtained from the orbit and the model atmosphere.

The daily partial derivatives are obtained in the same manner:

$$\frac{\partial \bar{\dot{\varepsilon}}}{\partial T_c} = \frac{\bar{B}}{2\Delta t} \int_0^{\Delta t} \frac{\partial \rho}{\partial T_c} f(v) dt \quad + \quad \frac{\bar{B} \Delta Q}{2\Delta t} \int_0^{\Delta t} \frac{\partial^2 \rho}{\partial Q \partial T_c} f(v) dt \tag{4}$$

$$\frac{\partial^2 \bar{\dot{\varepsilon}}}{\partial T_c^{\,2}} = \frac{\bar{B}}{2\Delta t} \int_0^{\Delta t} \frac{\partial^2 \rho}{\partial T_c^{\,2}} f(v) dt \quad + \quad \approx 0 \tag{5}$$

where

$\bar{B}$ = true B, not the orbit fitted B

ΔQ = perigee height bias in orbit fit

Q = perigee height

$\frac{\partial \rho}{\partial T_c}$ = partial derivative of density with respect to T_c

$\frac{\partial^2 \rho}{\partial Q \partial T_c}$ = second partial derivative of density with respect to Q and T_c

The partial derivatives with respect to the perigee height are needed because of the orbit errors in determining perigee height, mainly due to small numbers of observations sometimes available for the orbit fits.

For each time step in computing the sums, the HASDM atmospheric model, J70MOD, computes

$$\rho, \frac{\partial \rho}{\partial Q}, \text{ and } \frac{\partial \rho}{\partial T_c}$$

using the standard Jacchia atmospheric model inputs from the orbital parameters and solar flux values.

An immediate second call to the J70MOD model, with $\Delta T_c = 0.01°\,\text{K}$ as input is then used to compute the partials:

$$\frac{\partial^2 \rho}{\partial T_c^{\,2}} = \frac{1}{\Delta T_c}\left(\left(\frac{\partial \rho}{\partial T_c}\right)_2 - \left(\frac{\partial \rho}{\partial T_c}\right)_1\right) \tag{6}$$

$$\frac{\partial^2 \rho}{\partial Q \partial T_c} = \frac{1}{\Delta T_c}\left(\left(\frac{\partial \rho}{\partial Q}\right)_2 - \left(\frac{\partial \rho}{\partial Q}\right)_1\right) \tag{7}$$

where the subscripts 1 and 2 represent the values from the first and second calls to J70MOD respectively.

The integral sums, used to compute the EDR partials, are then computed daily from

$$\int_0^{\Delta t} \frac{\partial \rho}{\partial T_c} \mathrm{f}(\mathrm{v}) dt = \sum_{j=0}^{M} \left[\frac{\partial \rho}{\partial T_c} \right]_j \mathrm{f}(\mathrm{v}_j) dt \tag{8}$$

$$\int_0^{\Delta t} \frac{\partial^2 \rho}{\partial Q \partial T_c} \mathrm{f}(\mathrm{v}) dt = \sum_{j=0}^{M} \left[\frac{\partial^2 \rho}{\partial Q \partial T_c} \right]_j \mathrm{f}(\mathrm{v}_j) dt \tag{9}$$

$$\int_0^{\Delta t} \frac{\partial^2 \rho}{\partial T_c^{\,2}} \mathrm{f}(\mathrm{v}) dt = \sum_{j=0}^{M} \left[\frac{\partial^2 \rho}{\partial T_c^{\,2}} \right]_j \mathrm{f}(\mathrm{v}_j) dt \tag{10}$$

where $M = 86400 / dt$

The above sums are computed on a daily basis corresponding to the same time interval as the daily EDR values. The partial derivatives of EDR cannot be computed at this time in the procedure because the true B and perigee height errors are not yet known in order to evaluate Equations (4) and (5).

The daily EDR and partial derivative values (Equations (3), (8), (9), and (10)) are written to file for each day of the orbit fit. Also included is the orbit-fitted B value. For our previous example, 6 daily sets of values are written to file for this orbit fit. The observation start time is then increased by 1 day, new observations are obtained, the orbit is then refit with the new data, and the EDR and partial derivative sums are computed again for the new 6-day period.

Daily EDR Computation

The daily EDR values are obtained from special averaging of the same day overlapping values obtained from the orbit fits. If the orbit fits were without any error, then different orbit fit EDR values for the same day would be identical. However, because of the error in the fits, the best method for obtaining an accurate daily EDR value is not a simple average. It is necessary to constrain the "average" daily EDR values to maintain conservation of energy over each orbit fit span. This means that the energy change over each fit span must be preserved since these were the real measured quantities when the orbit fits were obtained. The following approach was developed to conserve the orbit fit energy changes.

Corresponding $\Delta \mathcal{E}_i$ values are computed using the B_i and $\bar{\dot{\mathcal{E}}}_i$ values from each orbit fit (i). A cubic fit employing least squares with $\Delta \mathcal{E}_i$ from 5 consecutive orbit fits is then used.

Setting $\dot{\bar{\varepsilon}} = \mathrm{a} + \mathrm{b}t + \mathrm{c}t^2 + \mathrm{d}t^3$ (11)

Integrating: $\Delta\varepsilon_i = \mathrm{a}t + \frac{1}{2}\mathrm{b}t^2 + \frac{1}{3}\mathrm{c}t^3 + \frac{1}{4}\mathrm{d}t^4 \Big|_{t_0}^{t_i}$ (12)

Five $\Delta\varepsilon_i$ values are used in Equation (12) to determine the four coefficients a, b, c, and d using least squares. The resulting Equation (12) represents the best fit energy change over the interval spanning the five orbit fits, while preserving, in a smoothed sense, the energy change over each of the five fits. The least squares approach was used instead of just fitting a cubic equation with 4 orbit fits because of problems that occur sometimes from a bad orbit fit.

New daily EDR values, representing the span over five orbit fits, can now be obtained from Equation (11). This procedure is repeated by moving one day in time and selecting five orbit fits over the next time interval. This method produces overlapping smoothed daily EDR values. The next step is to average these smoothed daily $\dot{\bar{\varepsilon}}$ values. The best-represented $\dot{\bar{\varepsilon}}$ value for the day should be when the value is closer to the middle of the 5 fit spans rather than at the ends of the interval. Therefore, the following weighting scheme was employed using the weighting function:

$$\mathrm{f} = \frac{1}{2} + \frac{1}{2}\cos\theta \ , \quad (13)$$

where

$$\theta = 2\pi\left(\frac{t\text{-}t_{\mathrm{BEG}}}{t_{\mathrm{END}}\text{-}t_{\mathrm{BEG}}}\right) - \pi \quad (14)$$

t_{BEG} and t_{END} are the times of the beginning and end of the 5 fit span interval t_{x} respectively. Then using the above weighting function the average EDR for each *i* day is obtained from the overlapping EDR values as:

$$\dot{\bar{\varepsilon}}_{\mathrm{AVE}\,t_x} = \frac{\sum_{i=1}^{n} \dot{\bar{\varepsilon}}_{t_x} \mathrm{f}_i}{\sum_{i=1}^{n} \mathrm{f}_i} \quad (15)$$

when using *n* daily values to compute the average.

The above method is also used to compute the daily average partial derivative sums that were obtained on a daily basis previously described.

Daily Temperature Computations

The daily temperature change is computed from the daily EDR and EDR partial derivatives previously computed. The temperature change equation is

$$\bar{\dot{\varepsilon}}_{\mathrm{OBS}} = \bar{\dot{\varepsilon}}_{\mathrm{COR}} + \Delta \mathrm{T_c} \frac{\partial \bar{\dot{\varepsilon}}_{\mathrm{COR}}}{\partial \mathrm{T_c}} + \frac{1}{2} \Delta \mathrm{T_c}^2 \frac{\partial^2 \bar{\dot{\varepsilon}}_{\mathrm{COR}}}{\partial \mathrm{T_c}^2} \tag{16}$$

where

$\bar{\dot{\varepsilon}}_{\mathrm{OBS}}$ = observed daily EDR computed from daily averaging
$\bar{\dot{\varepsilon}}_{\mathrm{COR}}$ = corrected EDR based on the true B and the perigee height correction

$$\bar{\dot{\varepsilon}}_{\mathrm{COR}} = \frac{\bar{\mathrm{B}}}{2\Delta \mathrm{t}} \int_0^{\Delta t} \rho_{COR} \mathrm{f(v)} dt \tag{17}$$

where $$\rho_{\mathrm{COR}} = \rho + \frac{\partial \rho}{\partial Q} \Delta Q \;. \tag{18}$$

ρ = atmospheric model density
$\Delta \mathrm{Q}$ = perigee height correction
$\frac{\partial \rho}{\partial \mathrm{Q}}$ = partial derivative of density with respect to perigee height correction
$\bar{\mathrm{B}}$ = true (30 year average) B value

An additional density correction was required. It was found that computing daily accurate perigee heights was necessary for obtaining accurate daily density values. Errors in the orbit fitted perigee heights result from fits with a minimum number of accurate observations that are not evenly spread throughout the fit observation span. They can also occur from larger than expected observational errors, which occur from time to time. These perigee errors will then introduce B errors, and eventually computed density errors, from using wrong heights in the density model. At a perigee height of 400 km, an error of 1 km in perigee height will introduce an error of 1% in the fitted B value, which in-turn will produce a 1% error in the EDR and resulting computed daily density value. The first step in determining a perigee height correction for each fit is to determine the perigee height at the beginning of each day for the fit span. The perigee height is changing from revolution to revolution due to the longitudinal gravity anomalies, and is changing day to day from the precession of the argument of perigee. The perigee heights for each revolution within a day are least squares fit with a constant and linear term to determine the value for the start of each day. Once all the starting day values from the orbit fits have been determined for the entire 20 to 30-year period the correction values can be determined. This is done by predicting the perigee heights for a six-month period, and then computing the difference from the predicted values and the start of the day values for the six-month period. The differences are then least squares fit with a constant and linear term. The resulting residuals are the perigee height corrections are used to correct the daily temperature values. This process is repeated for every six-month period until the entire 20 to 30-year time period has been processed.

Another very important step in computing accurate temperature and density values is the determination of the "true" B value for a satellite. A procedure[8] has been developed to compute accurate "true" B values. Thirty years of fitted B values from differential orbit corrections are obtained and averaged. This average is then used as the "true" B value for the satellite. Comparing many "true" B values of the same satellite type show that the 30-year averaging gives values consistent to within 2-3%. Table 1 shows a comparison of these "true" B values with a computed B value for a number of different spheres. Note that there are 5 Taifun-1 calibration spheres all at the same altitude, and the "true" B differences have a standard deviation of only 0.3% for all 5 satellites. The dimensional B value is obtained from the given area and mass and the best-estimated C_D value for a sphere at the orbit perigee height. The difference between the "true" B and dimensional B is shown as B-Btrue in percent error from the "true" B. Since the C_D is only accurate to approximately 10% the B-Btrue error is listed as ~10%. The B-Btrue values are all well within the ~10% C_D error, and are very close to a 3% accuracy value. This means that the Jacchia atmosphere being used in the orbit fits does not have a long-term bias greater than 3%.

SAT	Name	B_30Yr	Q Ht	Inc	Cd	Radius	M	Cd A/M	B-Btrue	B-Btrue		Ref
			km	deg		m	kg		%	Error		
02183	AE-B Exp. 32	0.00592	280	65	2.10	0.445	225	0.00581	**-1.9**	+- 10 %		9,12
5 Sats	Taifun-1 (Below)	0.01119	400	83	2.20	1.004	600	0.01160	**3.7**	+- 11 %		10
00011	Vanguard 2	0.05039	560	33	2.30	0.254	9.39	0.04965	**-1.5**	+- 10 %		11
05398	Rigidsphere 2	0.06098	775	88	2.35	0.560	37	0.06257	**2.6**	+- 10 %		9
02909	Surcal 150B	0.19578	850	70	2.40	0.203	1.55	0.20046	**2.4**	+- 10 %		9,14
02826	Surcal 160	0.19279	850	70	2.40	0.254	2.48	0.19615	**1.7**	+- 10 %		9,14
00900	Calsphere 1	0.24239	1025	90	2.40	0.180	0.98	0.24928	**2.8**	+- 10 %		9,14
	Taifun-1											
07337	COS 660	0.01120	405	83								
08744	COS 807	0.01117	405	83								
12138	COS 1238	0.01115	415	83								
12388	COS 1263	0.01121	385	83								
14483	COS 1508	0.01121	390	83								
	Ave	0.01119										
		+- 0.3 %										

Table 2. "True" B values compared with dimensional B values obtained for a number of spherical satellites. The references for the sphere dimensions are listed under the Ref column.

Once the true B has been determined the daily-corrected EDR value is computed from the orbit fit data. The ΔQ values for Equation (18) above are obtained daily as described earlier.

The values for input to Equation (16) consist of the computed daily average EDR value $\bar{\dot{\varepsilon}}_{OBS}$, the daily computed (modeled) EDR value $\bar{\dot{\varepsilon}}_{COR}$, and the daily average partial derivatives $\frac{\partial \bar{\dot{\varepsilon}}_{COR}}{\partial T_c}$ and $\frac{\partial^2 \bar{\dot{\varepsilon}}_{COR}}{\partial {T_c}^2}$ obtained from the same averaging method as with $\bar{\dot{\varepsilon}}_{OBS}$.

The quadratic Equation (16) is solved for ΔT_c, with the root closest to the linear solution selected as the value.

This ΔT_c value represents the daily temperature difference between the modified Jacchia model J70MOD and the observed temperature computed from the observed daily EDR value.

Daily Density Computations

An average daily density value can be obtained from the daily ΔT_c value using the following technique. The local solar time of perigee, perigee latitude, and perigee height are used for the density computation. These values, along with the daily ΔT_c, the solar flux daily $F_{10.7}$, average $\bar{F}_{10.7}$, and 3-hr geomagnetic index a_p value at the beginning of the day, are used in the Jacchia model to compute the density for time 00:00 UT at the beginning of the day. Since the ΔT_c is an average value over the day, the density must reflect this. Therefore, density values are computed every 3 hours from –12 hours to +12 hours of 00:00 UT using the different 3-hr a_p values for the same perigee local solar time, latitude, and height. These values are then averaged to obtain the daily density value. In addition, a reference perigee height is selected based on the perigee heights obtained over many years of data. The daily density value is then also computed for this reference height. This essentially removes the density variation due to height variations over an argument of perigee precession cycle. This method for using a reference height works well if the apogee height is at least 1000 km so that the great majority of the orbit change over the years lowers the apogee height and leaves the perigee height almost unchanged.

VALIDATION OF DAILY T_C AND DENSITY VALUES

Several different methods were used to validate the temperature and density computations. The first method compared the temperature values for two different satellites with temperature values computed by two other methods. Temperature values for year 2001 were computed for satellites 00060 and 04221 using the High Accuracy Satellite Drag Model Dynamic Calibration Atmosphere (DCA)[13] and the Energy Dissipation Model (EDM)[7]. The DCA model uses observations in an orbit differential correction fit with the temperature coefficient as a fitted parameter. The EDM model uses the reference orbit of the satellite to compute accurate EDR values, which in turn are used to compute the temperature values. The DCA model computes the temperature values every 6 hours, while the EDM method computes temperature values every one-half hour. These short time spans are possible only because of the very large amount of tracking data available for this time period. These two satellites were tracked every revolution during 2001 by at least one, and most of the time several, high accuracy radars. Figure 3 shows ΔTc values for satellite 00060 for part of the year. The DCA and EDM values were averaged on a daily basis about the beginning of each day for comparison with the currently described method (Average EDR). The agreement is excellent between this averaging method and the DCA method. The EDM method has excellent agreement with the phase but the amplitudes are somewhat higher during solar storms. This is most likely due to the half-hour resolution compared with the one-day resolution for the Average EDR method, even though the EDM values were averaged over a day. All three methods show excellent agreement for the ΔTc values during periods excluding major storm times.

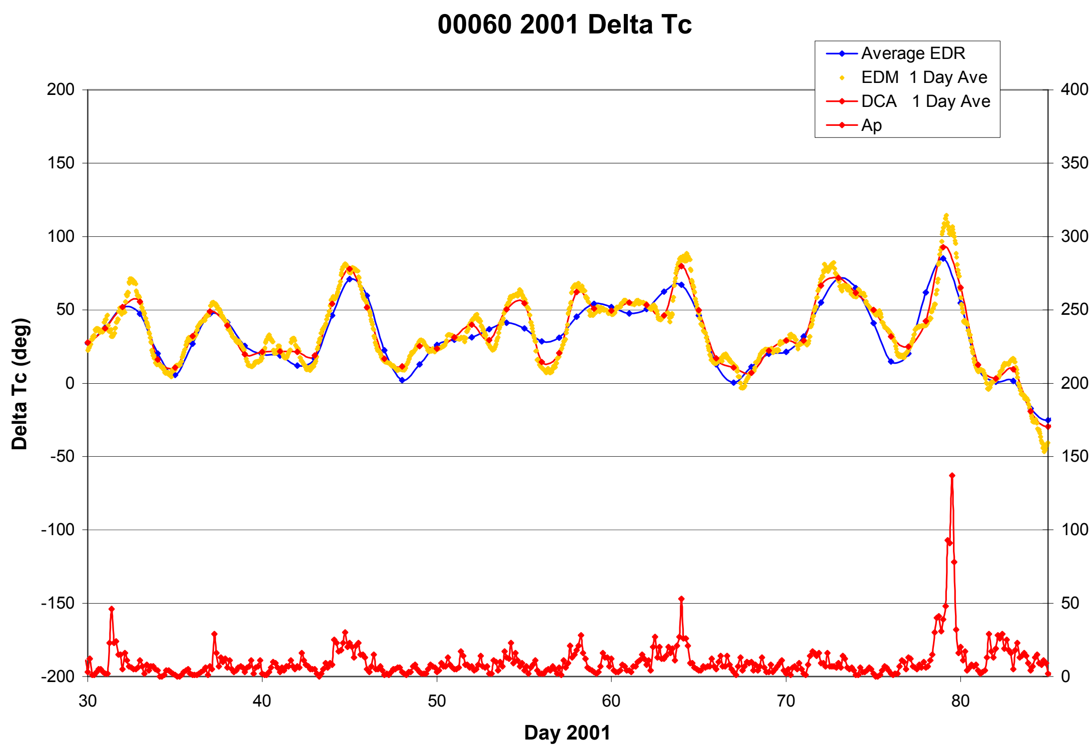

Figure 3. ΔT_c values obtained using satellite 00060, Explorer 8, from day 30 through 85 of 2001.

The second method used for validation consisted of refitting the orbits with the same observations per fit, but including the daily temperature values to correct the atmospheric model. If the temperature values account for all of the density variations from the original Jacchia model, then the resulting B values should be constant.

This procedure was done using the above two satellites. For computation of density points within each day a simple linear interpolation of the daily ΔTc values was used. The resulting fitted B values were then plotted and compared with the original B values obtained without any temperature corrections. The standard deviations of the original B values were about 15% for each satellite. The standard deviation values were reduced to less than 2% for both satellites. Figure 4 shows a plot of the B values. The "No dTc" plot values represent the original B values from the original orbit fits used to compute the daily temperatures, and the "dTc use" are the B values obtained when the daily Average EDR ΔTc values have been used in the Jacchia 1970 model. Note that even during the major storm periods the previously large B variations have been corrected out of the B values.

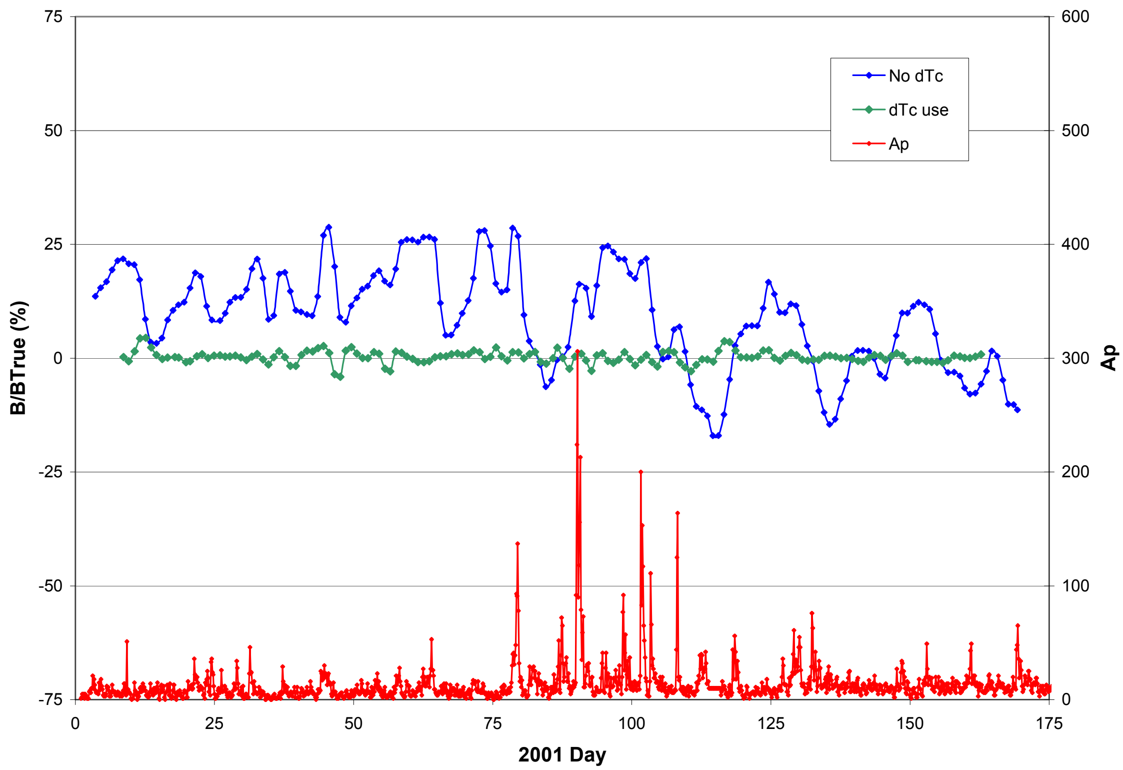

Figure 4. Ballistic coefficient variations obtained for satellite 00060 with and without daily temperature variations (dTc) applied in the modified Jacchia atmospheric model.

The third method used for validation consisted of comparing density values at perigee locations for multiple satellites with at least moderate eccentricity (apogee height at least 500 km greater than the perigee height, with the majority having apogee height at least 1000 km greater than the perigee height). The moderate eccentricity insured that the vast majority of the drag could be attributed to the perigee location. Over 25 satellites were used to obtained daily temperature and density values from 1969 through 2001, with over 250,000 daily values computed for the comparisons.

For each satellite processed from 1969 through 2000, approximately 100,000 radar and optical observations were available for orbit fitting. During the 1970s all the satellites were tracked by the accurate Eglin phased-array radar, and by the Navy's NAVSPASUR vertical radar fence across the entire United States. The orbit fits during this time period are very good when at least 5 days of data are used for a single fit. At the start of the 1980s, two of the many NORAD very accurate phased-array radars, plus the NAVSPASUR fence, were assigned to track each satellite. This meant that numerous very accurate radar observations were available daily for use in the orbit fits. During the 1990s additional tracking data was obtained to improve the orbit fit accuracy even more. From 1980 through 2002 the orbit fits on the satellites of interest are of excellent quality.

Once the daily temperature and density values were obtained for all the satellites, the perigee locations were compared on a daily basis. If the daily perigee location of one satellite was within 15 km altitude, 1.5 hours local solar time, and 15 deg latitude of another satellite, then a common mid-point was computed for the two satellites. This mid-point of height, local solar time, and latitude was then used along with each satellite's ΔT_c value to compute the daily average density value for each comparison satellite. The values were differenced to obtain the density error. These density errors were then used to compute a density standard deviation error by year. Over 8500 comparison pairs were obtained for the computations. Figure 5 shows the resulting standard deviation of the density values. It is immediately evident that during solar minimum times the errors are larger than during solar maximum times. This is expected since there is more atmospheric drag observability during solar maximum when the atmosphere has expanded compared to solar minimum times. Also evident is the improved orbit fit accuracy from 1980 to 2001 due to increased radar tracking.

The final results show that the density errors are less than 5 % for the last 20 plus years of data, and approach 2-3% error during the latest solar maximum period. This small density error using multiple satellites over multiple years demonstrates the validity of the EDR method of obtaining accurate daily temperature and density data.

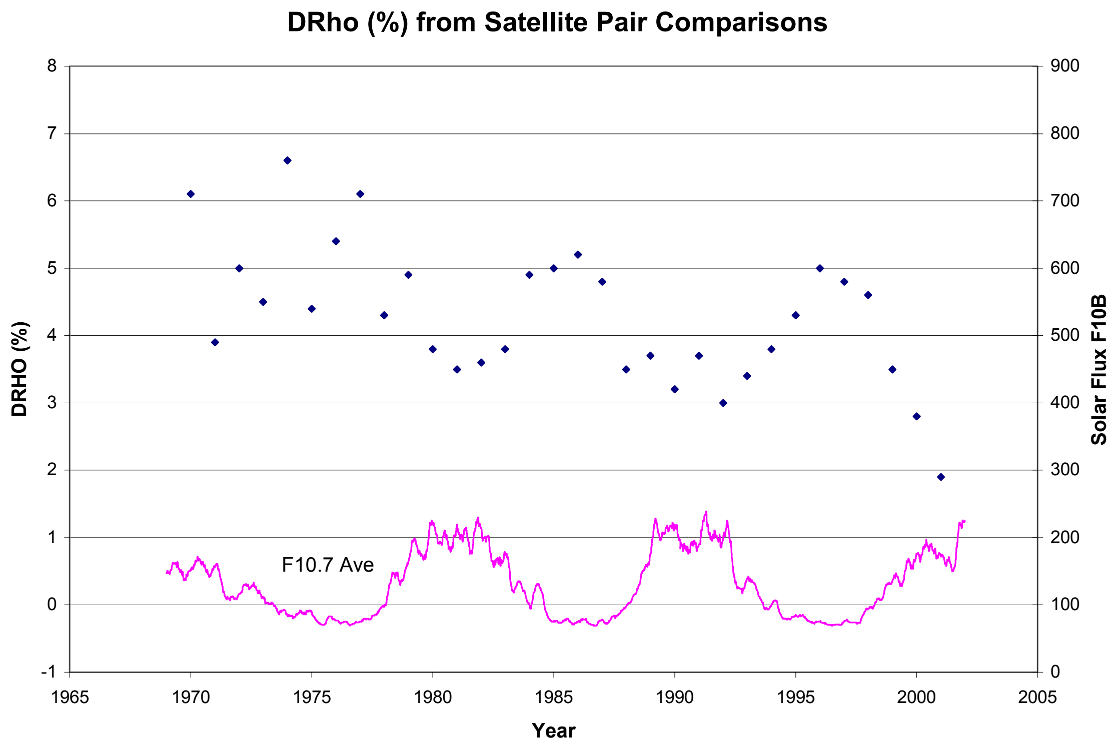

Figure 5. Standard deviation errors in density from comparison of values at the same perigee locations using over 8500 satellite pair values.

CONCLUSION

A method has been successfully developed to compute daily average density values from satellite energy dissipation rate computations. The method has been validated from density comparisons obtained using many satellites over 30 years of data. The accuracy of the density values was determined from comparisons of geographically overlapping perigee location data, with over 8500 pairs of density values used in the comparisons. The density errors are less than 4% overall, with errors on the order of 2% for values covering the latest solar maximum.

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